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The Slow Acting Relay

An Investigation of Eddy Current and Magnetism During Release

By Arthur Bessey Smith

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An Experimental Determination of the Space and Time Distribution of Foucault Current in a Cylinder and the Release Time of a Relay Armature

By Arthur Bessey Smith

PHYSICS

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The original work was done before June, 1926, at Northwestern University, Evanston, Illinois. The investigation was continued in the Research Laboratory of Automatic Electric Inc. until shortly before publication.

AN EXPERIMENTAL DETERMINATION OF THE SPACE AND TIME DISTRIBUTION OF FOUCAULT CURRENTS IN A CYLINDER AND THE RELEASE TIME OF A RELAY ARMATURE.

INTRODUCTION

This is an investigation of the relations existing between a solid mass of metal on an electromagnet, the magnetizing force applied to the magnet, and the delay of the armature to release.

The electromagnet used for this purpose is one which is very common in automatic telephony. It is the slow acting relay which occupies such a large place in the art that the fundamental laws governing its action ought to be more thoroughly known. It is believed that the relations discovered here are generally true. But the great importance of this particular structure is complete justification for the detailed study of it.

Relays of this type, Fig. 1, are made in two varieties, quick acting and slow acting. The former has its winding occupying all or nearly all of the winding space. The latter has part of that space filled with wire, the rest being filled with solid copper.

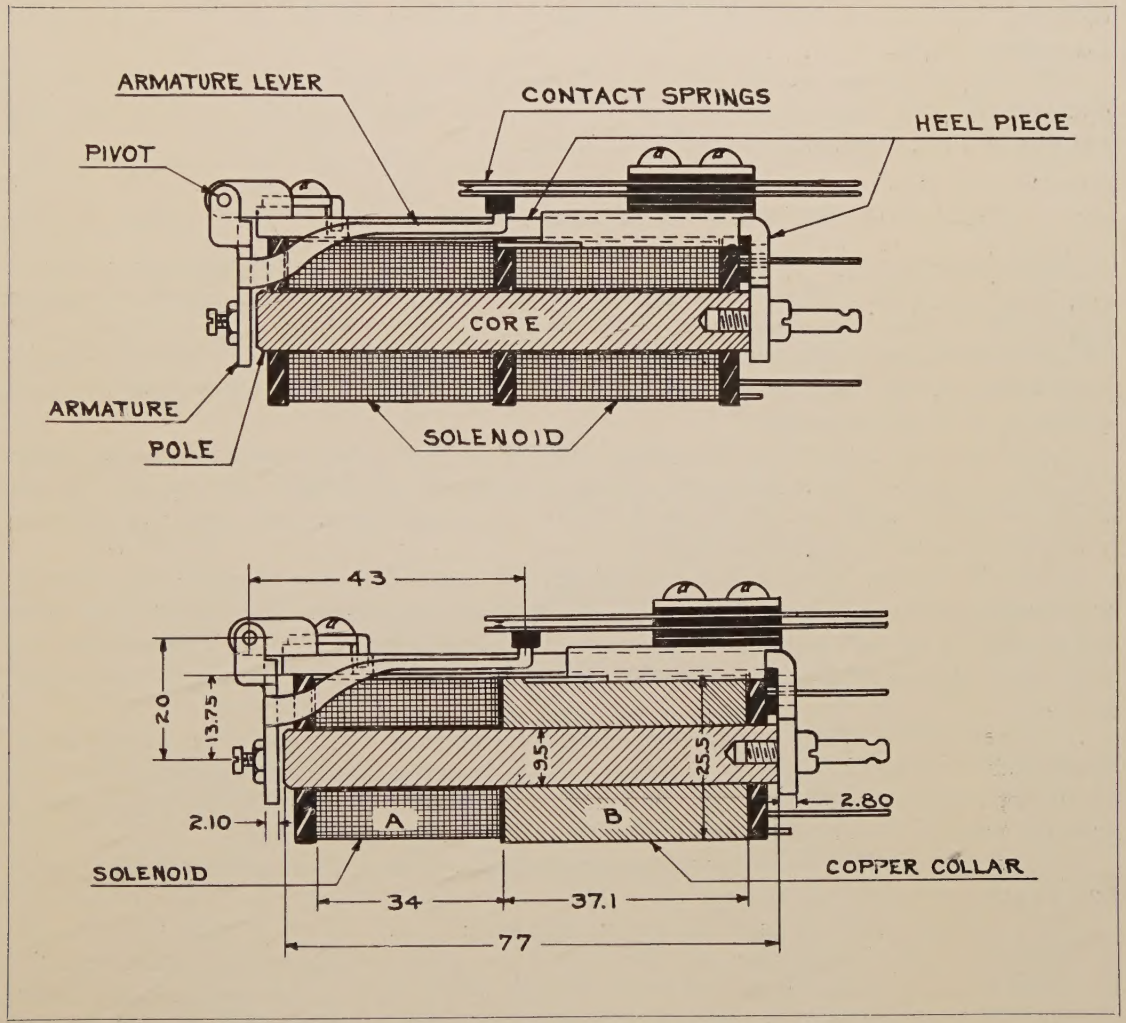


Fig. 1. Telephone Relays, Quick and Slow Acting

(Dimensions in mm.)

The magnetic circuit consists of an iron core, cylindrical, a flat return member (heelpiece) and an iron armature, pivoted on a pin. The springs which form the switch are mounted on the heelpiece, with a free length of about 38 mm.

The quick acting relay, having no copper collar, responds very quickly to energizing current in the coil. It also releases its armature very quickly after the current has been cut off. But the slow acting relay is much more deliberate. During the growth of current in the coil, the increasing magnetism which threads the copper collar induces a current of electricity in the copper, and the direction is such as to oppose the increase of the magnetism. After the current has been cut off, the decreasing magnetism induces a current in the copper, and the direction is such as to oppose the *decrease* of magnetism. Therefore the magnetism lasts longer.

Relays which must operate quickly, but release slowly, have the copper on the "heel" end; that is, away from the pole. This is the arrangement of the relay under study.

The total number of variables is so great, and their combination so much greater, that some of them had to be fixed during this investigation. The most uncertain is probably the armature, possessing masses at various radii, restoring force which varies not according to Hooke's law, and friction which has not yet been adequately studied. In this investigation, all things connected with the armature were fixed as much as possible, for the sake of learning about the magnet proper.

The solenoid or winding had 5070 turns of wire in a cylindrical space 34 mm. long, outside diameter 25.5 mm., inside diameter (hole) 10 mm. The copper collar had the same diameters but was 37.1 mm. long, and weighed 142.2 grams.

RELATION BETWEEN SOLENOID, COLLAR, AND POLE FLUX.

What happens when the current in the solenoid is cut off? Most electromagnets have a complicated structure. There is no simple magnetic curcuit, for there are distributed magnetomotive forces, fluxes, and reluctances, the latter variable with intensity of flux. There is need for simple expressions of over-all relations, which are more useful in many cases than expressions which are rigorously true through all the structure. The relations between the principal actors on the stage are sought.

At the moment just before the solenoid current is cut off, the solenoid, occupying region "A," Fig. 1, controls the situation, producing the pole flux which holds the armature against the restoring force, and also producing the flux in the copper collar, with which flux the latter will begin its work during release.

Then the current in the solenoid vanishes. All magnetism everywhere in the structure would vanish instantly if there were nowhere a restraining influence. There are two, the residual magnetism (or that property which causes it) and eddy currents. The former limits the drop of magnetism to a residual amount. The latter limits the rate of fall to the residual point.

In the short time after cut off, there is a quick change of authority. The solenoid ceases to act and the copper collar takes charge. The decreasing magnetism causes current in the copper, which current reacts on the magnetism, reducing the rate of fall so that after a time the magnetic flux reaches the value below which the armature releases.

The solenoid fixes the condition of the starting point. The inherent properties of the collar and its position control the finish.

The problems which we approach are the following:

(1) What is there about the collar and its location which conditions the "release time?"

(2) What is the time distribution of the flux caused by the Foucault current in the collar?

(3) How is the release time affected by the effective resistance of the collar to eddy current? The above is better expressed by the term "Foucault resistance," defined as the resistance taken in the direction in which the eddy current flows.

With these questions in mind, the writer first made a survey of the electromagnet.

The restoring force which acts on the armature is due to the contact springs. We are interested in this chiefly that we may know that it conforms to the usual practice and that it remains constant.

The values of the magnetic fluxes at selected places are useful, because we need to know the conditions which exist at the moment before the release process is initiated. Stronger magnetization of the iron results in longer release time; that is, the time from the cutting off of the solenoid current to the beginning of the movement of the armature away from the pole. For this reason varying values of ampere-turns were used in the solenoid.

It was also desired to know how much control the Foucault current in the copper collar could exert over the magnetic flux at the pole. The pole flux is responsible for the attraction of the armature, and if that flux falls below a certain point the armature will release.

The copper collar can control the pole flux to a certain extent and that control was measured.

Another thing which should be known is the manner in which the magnetism at the selected places decays after the solenoid current has been cut off. This is influenced by the Foucault current, chiefly in the copper collar. The resistance of the collar depends upon the material, its resistivity and shape. For these reasons decay curves of magnetism were discovered by experiment, using differing resistances of material in the collar space. This led to the conception of "Foucault resistance" which will be treated mathematically in connection with the results.

Lastly, the actual release time had to be measured, using as independent variable the resistance of the material in the collar space.

During the work of the research, unsuspected things were discovered. These are best told in the description of the experiments which led to their discovery.

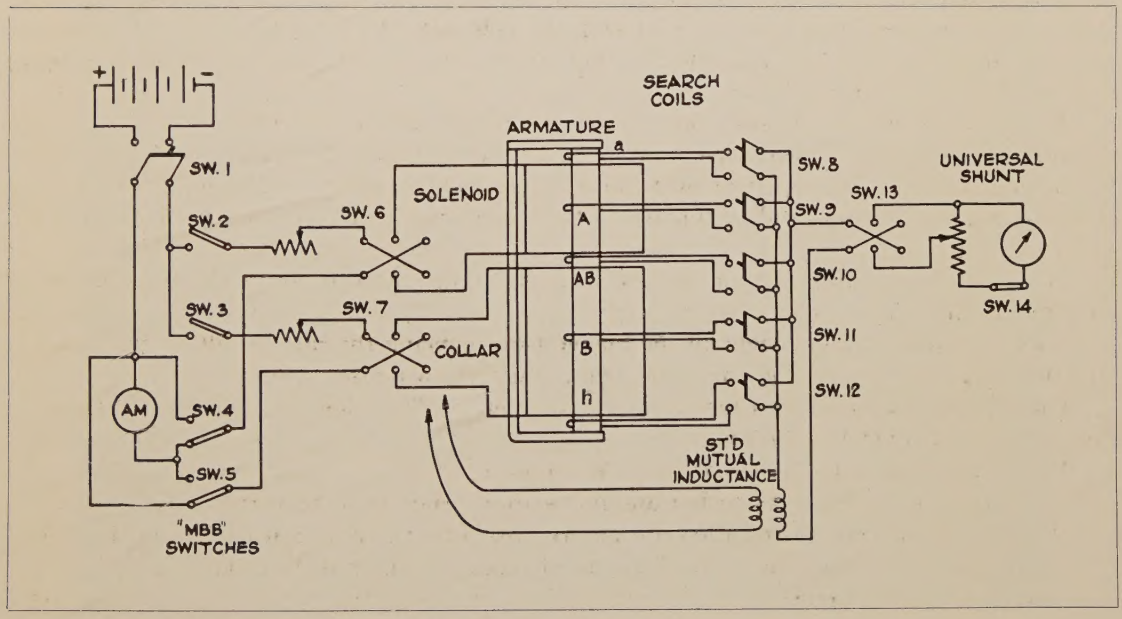


Fig. 2. Measurement of Magnetic Flux

RESULTS OF EXPERIMENT

STEADY CONDITIONS IN THE ELECTROMAGNET.

Restoring Force

The chief interest at present is in the release condition, the armature being close to the pole.

Table A

Air gap mm.....	zero	0.15	0.25 mm
Force at gap.....	215	68.8	54.8 grams
Force at springs.....	100	32	25.5 grams

Flux Distribution.

Data were secured from 104 ampere-turns, to 609 ampere-turns, but the lower values were not used. In the table below, NI means ampere-turns in the solenoid. The five search coils shown in Fig. 2 were located as follows: "a" at pole, "A" at center of solenoid, "AB" between solenoid and copper collar, "B" at center of copper collar, "h" at heel end of core. Each search coil had 20 turns, and closely fitted the core. Fig. 3 shows the relay with solenoid and search coils. Fig. 4 shows the general arrangement, and Fig. 5, the switches and relay.

Table B—Initial Magnetic Flux in Maxwells

NI	*a*	*A*	*AB*	*B*	*h*
307	7248	8430	7941	6996	6120
408	8216	9663	9220	7974	6925
508	8859	10629	10166	8686	7328
609	9361	11374	10810	9170	7630

These data were secured by the usual ballistic galvanometer method, Fig. 2. Copper knife switches were used everywhere except for the two reversing switches Sw-6 and Sw-7, which were telephone switchboard keys to secure very quick action. The five search coil circuits were equalized in resistance so as to simplify calculations. Reversing switch Sw-13 enabled all deflections to be in the same direction. The standard of mutual inductance was used for calibration by connecting it to Sw-6 in place of solenoid "A." The iron of the relay was reduced to a cyclic state by repeated reversals for two reasons, to conform to the conditions

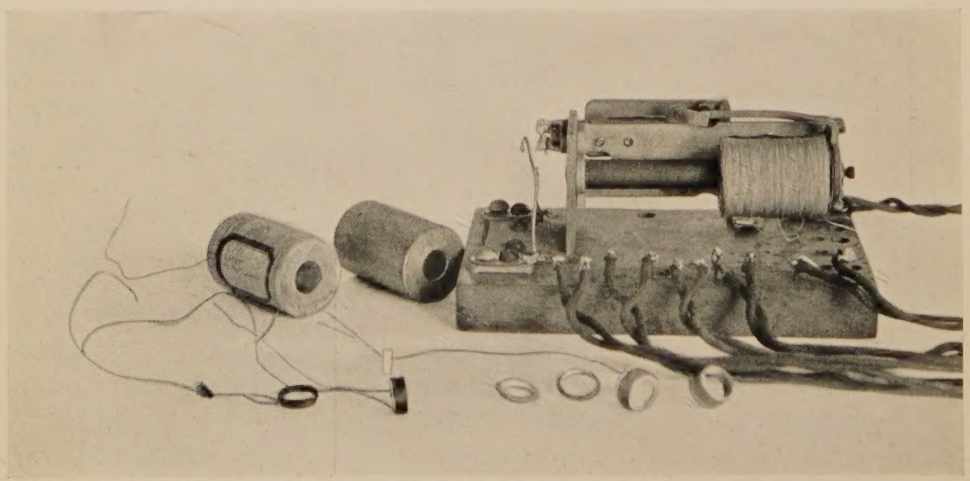


Fig. 3. Relay with Solenoid and Search Coils

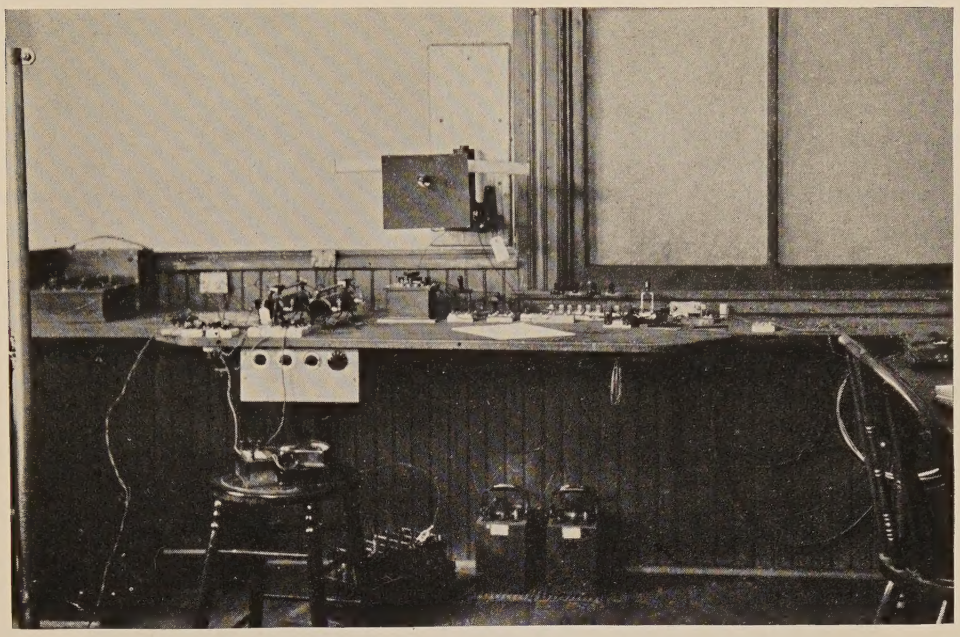


Fig. 4. General Arrangement of Apparatus

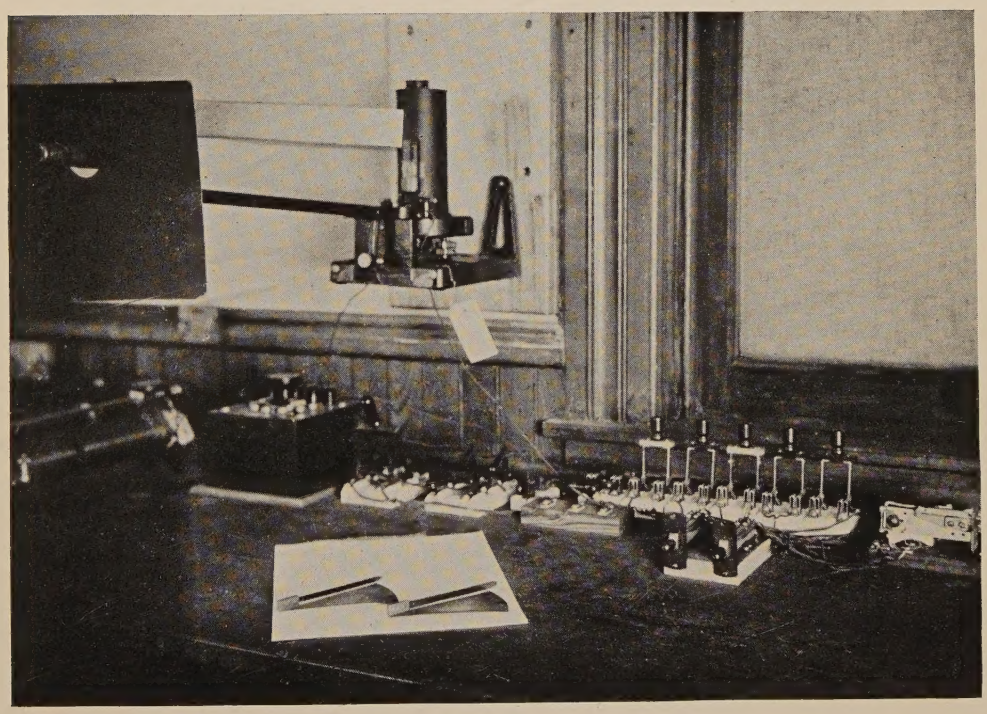


Fig. 5. Switches and Relay

of use, and to secure uniformity of results. Fluxes were calculated by the relation

$$\phi = \frac{10^8 R}{2n}\theta K \quad \ldots\ldots (1).$$

in which R = total resistance of search coil circuit, n = number of turns in search coil, θ = deflection of galvanometer, and K = galvanometer factor.

The galvanometer factor, K, was measured by the discharge from a known mutual inductance coil

$$K = \frac{2m}{R}\frac{I}{\theta} \quad \ldots\ldots (2).$$

in which m = mutual inductance in henries, R = total resistance of secondary circuit, I = current in primary in amperes, and θ = deflection of galvanometer.

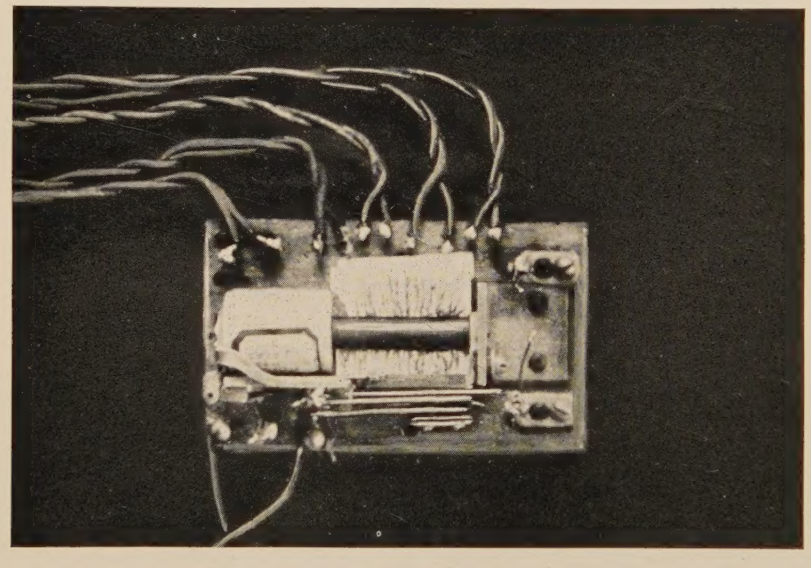


Fig. 6. Magnetic Leakage

In the collar space, the leakage of magnetism from core to return member was practically uniform; that is, from the "AB" search coil at the end of the solenoid, to the "h" coil at the heel end. This was held to be sufficient ground for judging the magnetism enclosed in the collar by the value measured at the middle of its length, the "B" coil.

Magnetic leakage between core and heelpiece is perpendicular to the axis of the core, and the plane of this leakage flux is perpendicular to the surface of the heelpiece. This was discovered by sprinkling iron filings on a cardboard fitted between core and heelpiece, Fig. 6. This test was suggested by the discovery described in the following paragraph.

All the magnetic flux which can produce really useful eddy currents is in the core itself. This was discovered by the use of five search coils, built up into one concentric unit, all coils having their corresponding ends in the same plane. Each coil had 20 turns, a single layer of No. 39 B & S single silk copper wire, each coil being about 3.5 mm long. The other facts are as follows:

Table C

Coil No.	*Diam.*	*Resistance.*	
1	10 mm	2.58 ohms	
2	15.5	3.27	
3	17.8	3.99	With leadout
4	21.8	4.81	wire 17 cm. long.
5	25.5	5.37	

In the following table, *NI* means ampere turns in the solenoid.

Table D—Flux in Various Areas

NI	*1*	*2*	*3*	*4*	*5*
307	6969	7052	6969	6969	6969
408	7994	8035	7994	7994	7994
508	8732	8773	8732	8732	8732
609	9183	9224	9183	9183	9183

An apparent paradox presented itself. There was leakage all along the core, yet a search coil of large diameter cut no more flux than one of small diameter. This led to the iron filing test, given above.

Two facts emerge. As regards current flowing in the collar and *around* the core, we can assume that all the magnetic flux is in the core. As regards the leakage flux, it doubtless causes current to flow in the material of the collar, but not encircling the core. The effect of this on the release time is not considered.

Foucault Resistance of Collar.

Consider Fig. 7 a uniform hollow metal cylinder of length u, inside radius v, and outside radius W. Further, consider an elementary shell of this cylinder of very small thickness Δw, resistivity ρ, and radius w.

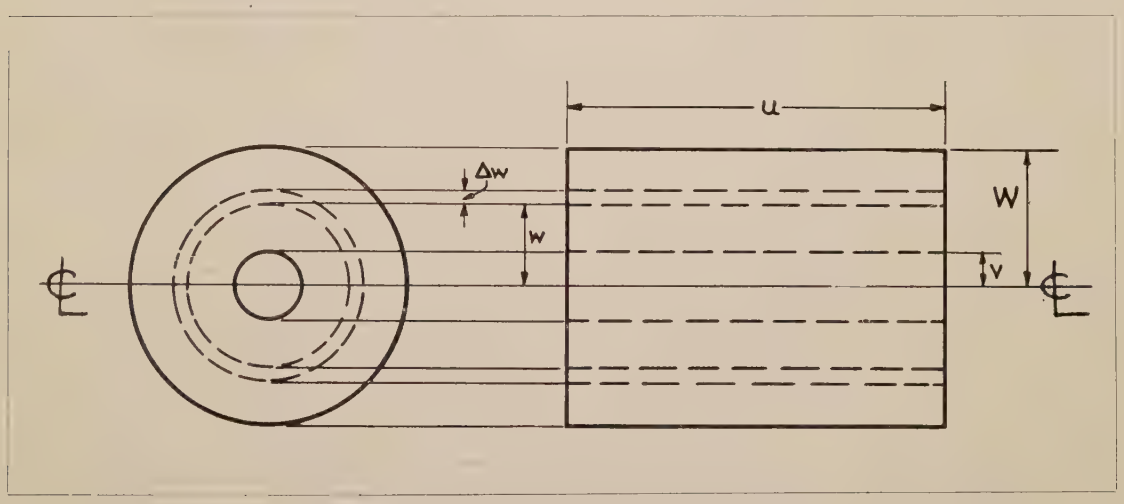


Fig. 7. Foucault Resistance of Collar

This shell may be considered as a single turn of metal. Let e be the emf developed in it by the changing magnetism, all of which passes through the hollow cylinder.

Let R = Foucault resistance of the collar

ΔR = resistance of elementary shell when taken in the direction around the axis.

Then

$$\Delta R = \frac{\rho 2\pi w}{u \Delta w} \quad \ldots\ldots (3) \qquad \Delta I = \frac{e}{\Delta R} - \frac{eu \Delta w}{2\pi \rho w} \quad \ldots\ldots (4).$$

$$\text{Total current} = \sum \Delta I = \frac{eu}{2\pi \rho} \Sigma \frac{\Delta w}{w} \quad \ldots\ldots (5).$$

Pass to the limit

$$I = \frac{eu}{2\pi \rho} \int_v^w \frac{dw}{w} = \frac{eu}{2\pi \rho} log_e \frac{W}{v} = \frac{eu}{2\pi \rho}\ 2.3026\ log_{10} \frac{W}{v} \quad \ldots\ldots (6).$$

So that

$$I = e \times 0.3665 \frac{u}{\rho} log_{10} \frac{W}{v} \quad \ldots\ldots (7).$$

The Foucault resistance is the reciprocal of the coefficient of the e.m.f. "*e*" Thus

$$R = \frac{\rho}{u\ log_{10} \frac{W}{v}} \frac{1}{0.3665} = 2.728 \frac{\rho}{u\ log_{10} \frac{W}{v}} \quad \ldots\ldots (8).$$

In order to vary the Foucault resistance of the collar, there were used in addition to copper, (i) a brass cylinder of the same dimensions and (ii) a 6000-turn coil of wire, which was sometimes closed on itself with zero resistance and at other times with a resistance equal to its own; namely, 282 ohms.

The following values were calculated for copper and for brass:

Copper $= 1.724 \times 10^{-6}$ *ohm per cm.*^{3} $R = 3.247 \times 10^{-6}$ *ohm.*

Brass $= 7.8 \times 10^{-6}$ *ohm per cm.*^{3} $R = 14.64 \times 10^{-6}$ *ohm.*

In absence of reliable values of resistivity for brass, the above figure was chosen because the brass used was known as "free cutting."

The Foucault resistance of the 6000-turn coil is the resistance of a single turn which will replace the coil. It was calculated from $R = \frac{r}{N^2}$ in which r = measured resistance of coil and N = number of turns

The derivation is as follows:

The total e.m.f. developed in the coil by flux change is Ne.

The resulting current is $\frac{Ne}{r}$

The ampere turns are $N \frac{Ne}{r} = \frac{N^2 e}{r}$

The resistance of a single turn which could replace the coil is the reciprocal of the coefficient of "*e*," that is $\frac{r}{N^2}$ ..... (9).

The 6000 turn coil has a resistance of 282 ohms, so that $R = \frac{282}{6000^2} = 7.833 \times 10^{-6}$

The same coil closed on 282 ohms has twice the Foucault value, that is 15.666×10^{-6} ohm.

The current density in the above coil is uniform throughout the volume of the coil, whereas it varies from core to outside in the solid metal collar. This difference was for the time being neglected.

The Foucault resistance which must be considered in relay action is the joint effect of collar resistance and core resistance. Because the core is iron and because it is solid to the center, I found no means of calculating its Foucault resistance. Consequently, I was forced to discover

it (at least approximately) from the experimental curves. Function (44) indicates that the time required for the flux to fall to any value is inversely proportional to the Foucault resistance. Hence, values of time for a given flux were taken from the "core alone" curve and the copper curve of Fig. 14 and ratios computed as follows:

Table E

Magnetic Flux	*Time*		*Ratio* $z=\frac{T}{t}$
	T for copper	*t for core alone*	
4000	0.119	0.008	14.875
3500	0.148	0.01	14.800
3000	0.187	0.014	13.357
2500	0.239	0.018	13.278
2000	0.310	0.025	12.400
			Average $z=13.742$

Let R = resistance and G = conductance of collar

r = resistance and g = conductance of core

Then $\frac{G+g}{g}=\frac{T}{t}=z$ so that $g=\frac{G}{z-1}$

For the copper collar $R=3.247\times10^{-6}$ so that $G=307977$

Then $g=\frac{307977}{13.742-1}=24170.9$ and $r=41.373\times10^{-6}$ ohm.

The joint conductance $G_0=307977+24170.9=332148$ mhos.

The joint resistance is the reciprocal $R_0=3.0107\times10^{-6}$ ohm.

Control of Collar over Pole Flux.

In order to discover how much magnetism can be produced by current in the collar, at the pole, where it will affect the armature, a 6000 turn coil was used in that space. Current sent into it caused magnetism at the pole, which was measured by the search coil-and-galvanometer method. I have expressed this control by a quantity "P" which might be called "power" and is defined by

$$P=\frac{\phi}{NI} \qquad (10).$$

in which

ϕ = flux at pole

NI = ampere turns in collar

Thus P is "maxwells per ampere turn."

Table F

Ampere turns NI	*Flux at Pole*	*Flux per Ampere-turn*	*Flux in Collar*
30.9	392	12.68	726
61.5	1161	18.87	1889
92.1	2013	21.86	3126
123	2714	22.12	4231
153	3416	22.32	5243
183	4029	21.98	6153
243	4961	20.41	7476
304	5642	18.58	8594
364	6194	17.04	9497
424	6596	15.56	10223
483	6840	14.16	10730
542	7047	13.00	11138
602	7254	12.06	11496
662	7374	10.89	11799
721	7477	10.37	12102

These results are plotted as curves in Fig. 8. Attention is called to the relatively low value of NI for which P has its maximum. It is approximately $NI = 140$ ampere-turns, where $P = 27$ maxwells per ampere turn, and $\phi = 3100$ maxwells.

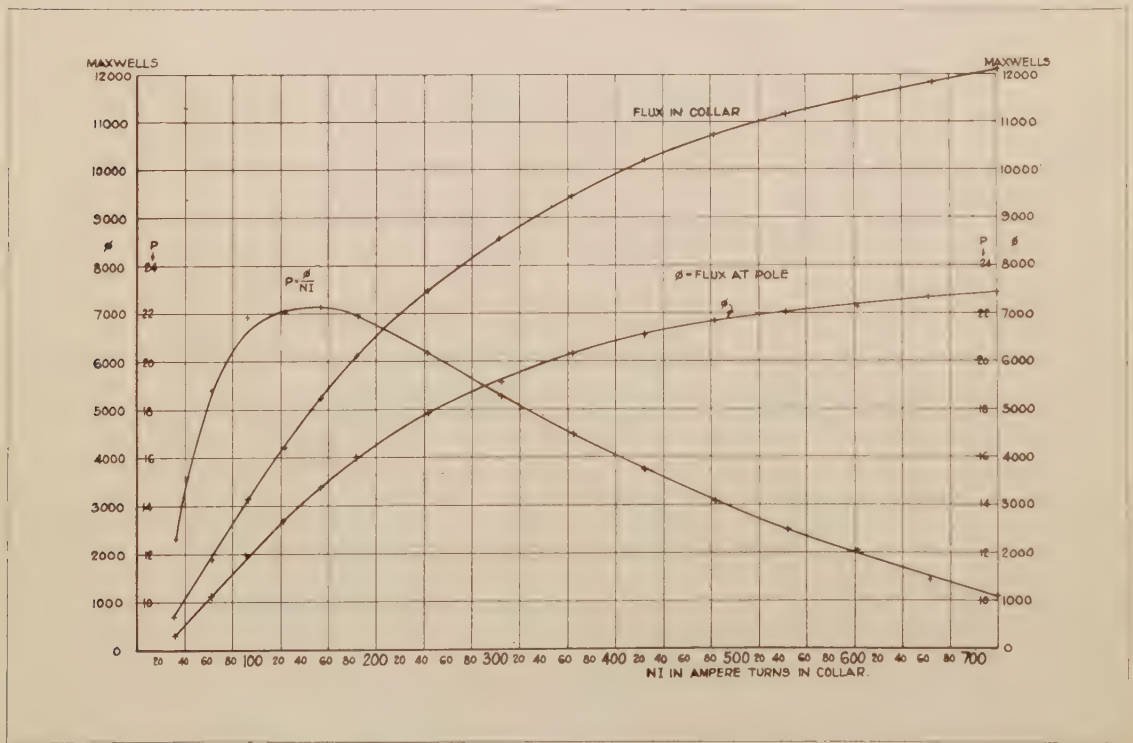


Fig. 8. Control of Collar over Pole Flux

DECAY OF MAGNETIC FLUX AT POLE AND COLLAR

The decay of magnetism with time, after the cutting off of the solenoid, was measured as follows: The galvanometer was connected to the search coil during a definite interval of time, and the flux change during this interval was subtracted from its known initial value.

The procedure was as follows:

1. Bring the magnet to a cyclic state.
2. Measure initial flux by reversal.
3. Measure the drop to the residual flux.
4. Restore to cyclic state.
5. Deliver current from search coil to galvanometer for a predetermined short time.

This was repeated for each of the points on the curve.

The curcuit is shown in Fig. 9. Only one search coil is shown, the others being wired as in Fig. 2, with resistances equalized. Other features are similar to Fig. 2. The two balls fall side by side, the left one opening Sw-4 to cut off the solenoid, the right one opening Sw-5 after the desired time interval. The shunt ratio used was great enough to prevent any appreciable change in the damping of the galvanometer after the search coil was cut off.

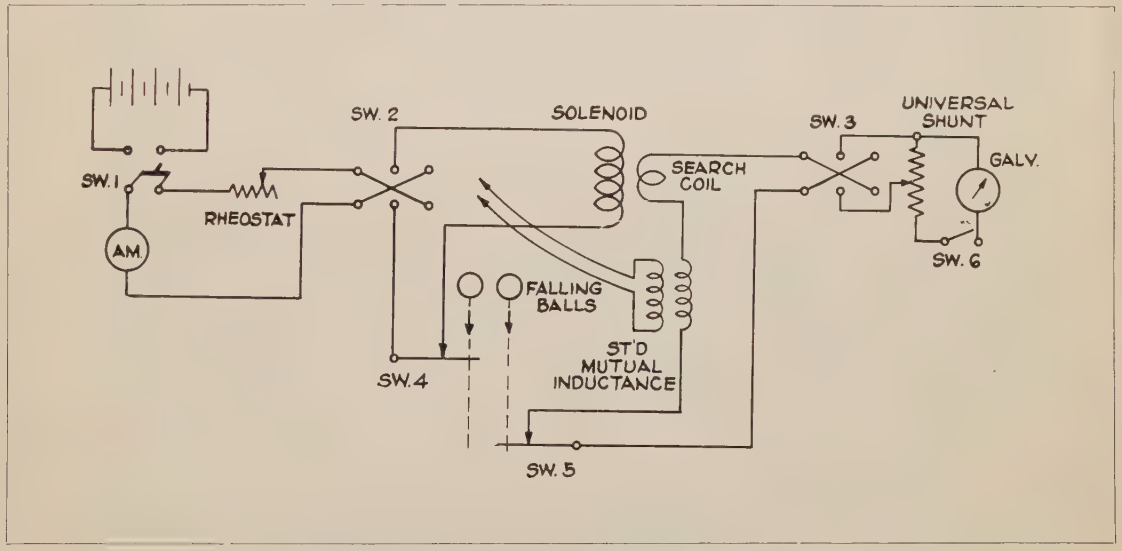


Fig. 9. Measurement of Decay of Magnetism

Chronometer Fall Device

The plan of chronometer contacts is shown in Fig. 10. The main spring "2" is mounted on a pivoted metal bar but insulated from it. On top of it is a damping spring "4" with a felt pad to absorb vibrations. The back contact "1" is a pivoted bar at the right, which is acted upon by a spring pulling toward the right. The motion is prevented by a small arm 5, whose insulated end rests on a pin in the pivoted bar 3. The front contact 3 is the pivoted bar itself. All three contacts are connected to the circuit by flexible stranded copper conductors to insure uniformity of resistance.

When the falling ball strikes the trip lever 6, the right end of 3 rises and causes the transfer of the circuit of 2 from 1 to 3, as shown in the little sketch at the right. This make-before-break arrangement was used for testing purposes and to make the apparatus of general use.

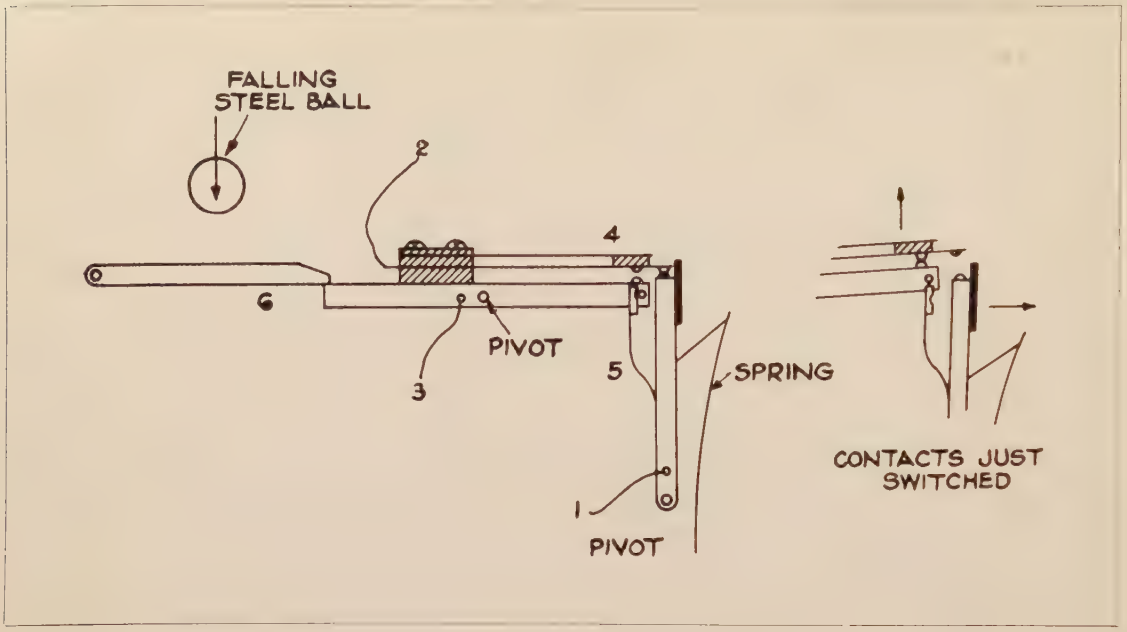


Fig. 10. Plan of Chronometer Contacts

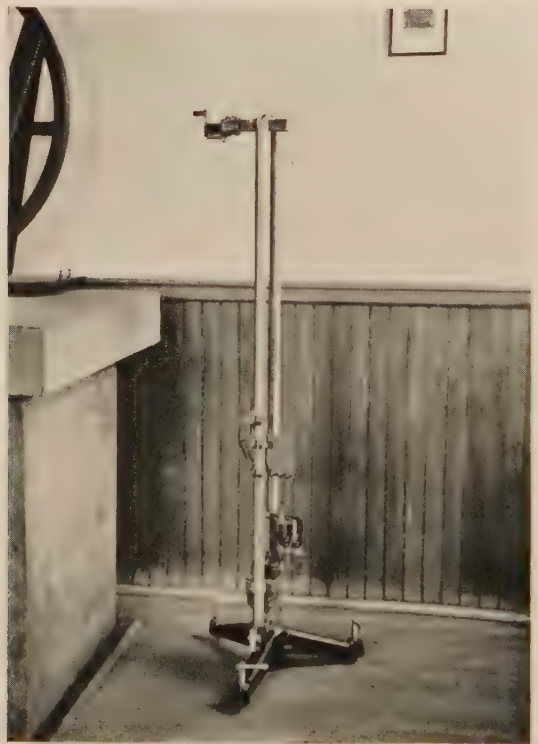


Fig. 11. Chronometer Fall Device

The complete chronometer fall device, Fig. 11, consists of a seamless drawn steel tube frame (2 side members) standing on a heavy tripod base and carrying the ball release at the top. The latter consists of a pivoted frame upon which rest the two balls. The frame is held against a stop by a toggle spring. A spring driven hammer, released by an electromagnet, drives the ball frame down and out of the way of the balls, at a speed greater than that due to gravity so that the balls have a free fall. The balls, after operating the contacts, fall into cloth bags, shown more clearly in Fig. 12. This shows one of the sets of contacts "set" ready for action. Fig. 13 shows the contacts released or operated.

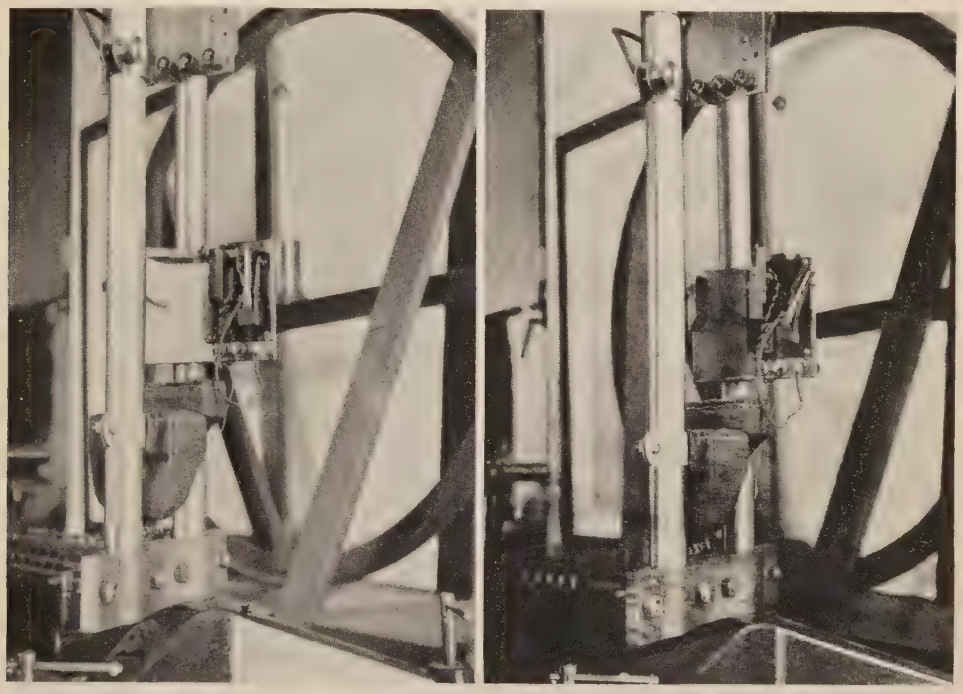


Fig. 12. Contacts of Chronometer (Set) **Fig. 13. Contacts of Chronometer (Released)**

The balls used were of steel, 25.4 mm in diameter and weighing 66.715 and 66.740 grams respectively.

The above method was suggested by the device due to Dr. Paul E. Klopsteg, modified to use two balls instead of one.*

For most of the measurements the shortness of the intervals (from 0.003 to 0.150 sec.) demanded the use of two balls; but for the longer intervals (say 0.200 sec.) one ball, with contact on the release, was used.

Flux Decay Curves by Experiment

In the following tables, all fluxes are in maxwells. The solenoid, at the time of cut off, carried 508 ampere turns. "Core" means no collar at all. "6000-T half" means the use of that coil as collar, closed through a resistance equal to itself. "6000-T" means the same coil, dead short circuited. "Copper" is the regular copper collar.

*A New Chronoscope and Fall Apparatus, by Paul E. Klopsteg, Journal of Experimental Phychology, Vol. 2, No. 3, August 1917, pp. 253 -263.

Table G—Decay of Magnetism in Collar Space

	Flux remaining at time "t"			
Time Seconds	*Core*	*6000-T half*	*6000-T*	*Copper*
zero	8679	8679	8679	8686
0.003	4757	7705	8199	
0.005				8517
0.010	3335	6371	7358	7892
0.015	3195	5608	6869	7850
0.025	2026	4591	6326	7170
0.050	1388	2722	4176	5867
0.100	1066	1709	2590	4446
0.150	1062	1318	2342	3484
0.200				3014
0.300				2087
0.400				1764
0.458				1722
∞	1002 maxwells for all collars			

Table H—Decay of Magnetism at End of Pole (Armature)

	Flux remaining at time "t"			
Time Seconds	*Core*	*6000-T half*	*6000-T*	*Copper*
zero	8859	8859	8859	8859
0.003	4600	5568	5860	
0.005				6208
0.010	2682	4408	5332	5960
0.015	2553	4323	4725	5558
0.025	1585	3428	4410	5092
0.050	1077	2204	3004	4428
0.100	921	1319	1882	3216
0.150	918	1242	1391	2577
0.200				2102
0.300				1631
0.400				1423
0.458				1352
∞	778 maxwells for all collars			

The above results are plotted as curves in Fig. 14.

MATHEMATICAL EXPRESSIONS

The discovery of a mathematical expression to represent these experimental curves may be made along two paths.

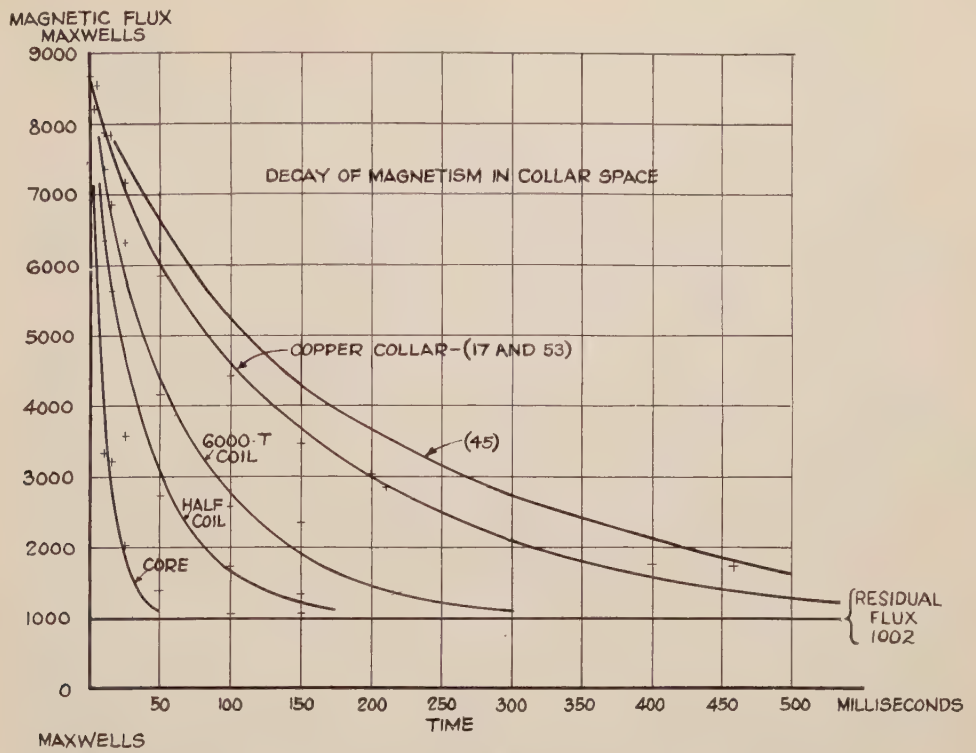
First, fit to the curve an expression empirically selected for good approximation.

Second, derive an expression from known fundamentals which will therefore take account of the structure of the relay.

The first method is the easier; the second method will come nearer to revealing what is happening during release.

Taking the first method, it is noted that the classical expression for the decay of current in the secondary winding of a transformer with an air core is a pure exponential.

$$i_2 = \frac{ME_1}{NR} e^{-\frac{S}{\Lambda} t} \quad \ldots\ldots\ldots\



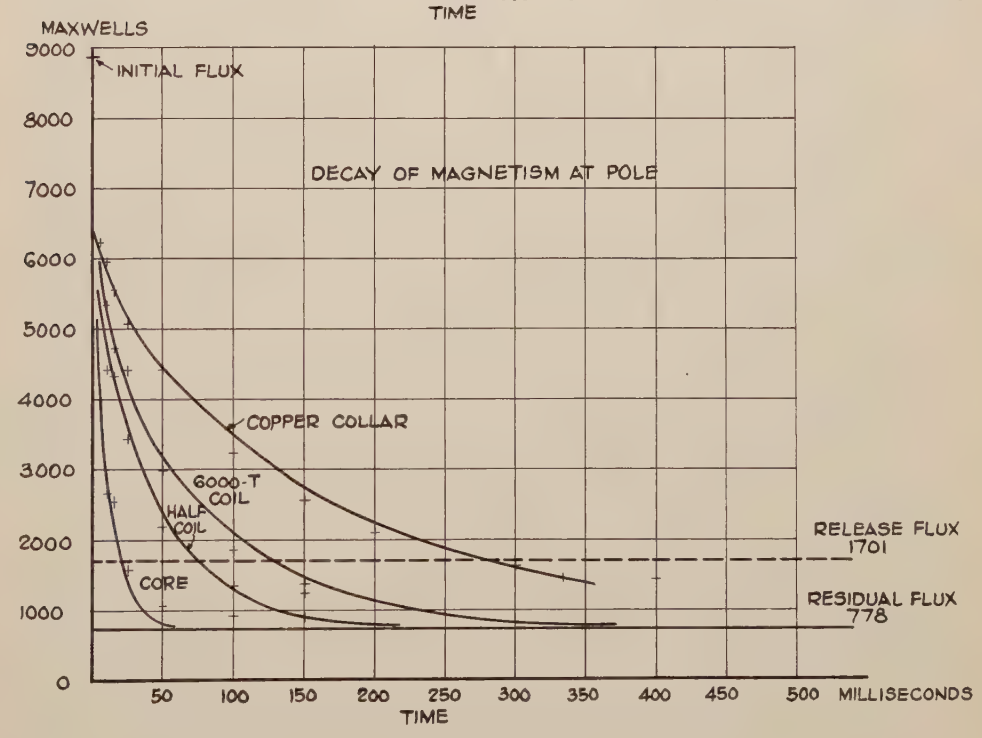


Fig. 14. Decay of Magnetism

in which E_1 = e.m.f. of battery in primary
M = mutual inductance
N = inductance of secondary
R = resistance of primary
S = resistance of secondary

Since magnetism is closely linked with current, it is reasonable to suppose that the decay of magnetic flux inside the copper collar should also be an exponential.

The application of a pure exponential plus a constant, to the curve, gave an expression which agreed at $t=o$ and $t=\infty$ and at one point between. A fairly close agreement was secured by adding a correction term, which decreased rapidly. The complete expression has the form

$$\phi = \Phi + M10^{-Qt} - D10^{-kt} \sin \frac{\pi t}{x} \quad \ldots\ldots (12).$$

ϕ = flux in center of collar at time t (in seconds)
Φ = residual flux
M = initial flux less residual flux
Q = index of exponential, computed by

$$Q = \frac{\log M - \log (\phi_x - \Phi)}{x} \quad \ldots\ldots (13).$$

All logarithms are to the base 10
ϕ_x = flux at time x
This exponential $\phi = \Phi + M10^{-Qt}$ ..... (14).
passes through $t=o$ $\quad t=x \quad t=\infty$

$$k = \frac{\log \left[u \sin \frac{\pi}{x} t_2 \right] - \log \left[v \sin \frac{\pi}{x} t_1 \right]}{t_2 - t_1} \quad \ldots\ldots (15).$$

$$D = \frac{u10^{kt_1}}{\sin \frac{\pi}{x} t_1} \quad \ldots\ldots (16).$$

u = desired correction at arbitrary time t_1
v = desired correction at time t_2 such that $t_1 < t_2 < x$

The procedure was to select x and calculate M and Q.
Then discover the error of (14) at t_1 and t_2.
Then calculate k and D.

To fit (12) to the decay of magnetism in the copper collar of Fig. 14, the following values were used.

Initial flux = 8686 maxwells
Φ = 1002 maxwells
x = 0.3 second
ϕ_x = 2087 maxwells

From these, $M = 7684$ $\quad Q = 2.8339$
The errors proved to be

$u = 550$ $\quad$ at $t_1 = 0.1$
$v = 403$ $\quad$ at $t_2 = 0.2$

From which were calculated

$k = 3.9482$ $\quad D = 1577$

The complete expression is

$$\phi = 1002 + 7684 \times 10^{-2.2451t} - 1577 \times 10^{-3.3307t} \sin 10.472t \quad \ldots\ldots (17).$$

The magnetic fluxes calculated are as follows:

Table J
Magnetic Flux

Time	*Experimental*	*Calculated*	*Error*
0	8686	8686	0
0.075	5050	5065	$\frac{1}{3}$ of 1%
0.1	4446	4369	−2%
0.15	3484	3440	$-1\frac{1}{4}$%
0.2	3014	2861	−5%
0.25	2500	2511	$\frac{1}{2}$ of 1%
0.3	2087	2266	8%
0.4	1764	1896	7.5%
0.458	1722	1722	0
0.5	1636	1607	−2%
∞	1002	1002	0

The second method of securing an expression for the decay of magnetism in the relay core inside the copper collar involves a complete analysis which reaches back to the behavior of iron, and the mathematical consideration of its permeability.

We start with the idea of a coil short circuited on itself, Fig. 15, for which the expression is

$$i = Ie^{-\frac{Rt}{L}} \quad \ldots\ldots (18).$$

I = initial current
R = resistance of coil
L = inductance of coil
t = time
e = base of Naperian logarithms.

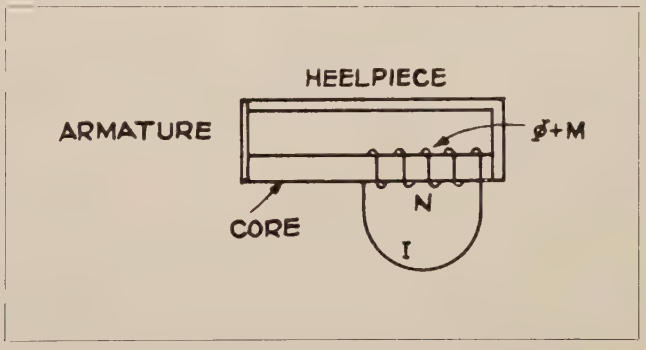


Fig. 15. Coil Short Circuited

This assumes that the inductance L is constant, which is far from true for an iron cored coil.

As is well known, the inductance of a coil depends upon the permeability of its core. This forces us at the start to discover the mathematical expression for permeability of iron as it exists in the electromagnet structure being studied.

The usual idea of permeability is obtained from the normal magnetization curve of iron and is expressed

$$\mu_3 = \frac{\beta}{H} \quad \ldots\ldots (19).$$

in which H = field intensity and β the flux density which results from bringing H from zero

to the desired value.

But during the release of a relay, the permeability which affects inductance is very different, as shown by the following line of reasoning.

While the magnetism is decreasing in the coil of Fig. 15, there is generated an e.m.f. thus

$$e = L\frac{\Delta i}{\Delta t} \quad \ldots\ldots (20).$$

This may also be expressed

$$e = 10^{-8} N \frac{\Delta \phi}{\Delta t} \quad \ldots\ldots (21).$$

Now $\phi = S\beta$ in which S = cross sectional area of core so that $\phi = S\,\Delta\beta$ and (21) becomes.

$$e = 10^{-8} NS \frac{\Delta \beta}{\Delta t} \quad \ldots\ldots (22).$$

Equating these two values of e (20) and (21)

$$\text{we have } L\frac{\Delta i}{\Delta t} = 10^{-8} SN \frac{\Delta \beta}{\Delta t} \quad \ldots\ldots (23).$$

Since the increment of time is the same on both sides of the equation, we may multiply it out. At the same time, divide by Δi so that we secure a value of inductance.

$$L = 10^{-8} SN \frac{\Delta \beta}{\Delta i} \quad \ldots\ldots (24).$$

Now, to relate the current i to field intensity H, we introduce a fiction. We cannot know the real value of H by the classic expression

$$H = \frac{0.4\pi NI}{c} \quad \ldots\ldots (25).$$

N = *turns in coil* I = *current in amperes* c = *length of magnetic path (cm.)*

because the flux is not confined to one path but spreads out by several leakage paths. But we may call "H" the *apparent* magnetic field intensity, and proceed.

Solve (25) for I and express it as an increment

$$\Delta i = \frac{c}{0.4\pi N} \Delta H \quad \ldots\ldots (27).$$

Put this into (24) securing

$$L = 4\pi 10^{-9} \frac{SN^2}{c} \frac{\Delta \beta}{\Delta H} \quad \ldots\ldots (28).$$

The quantity $\frac{\Delta \beta}{\Delta H}$ is the kind of permeability with which we are concerned. It is graphically illustrated in Fig. 16 and has been termed "differential permeability."* While the apparent field intensity is decreasing from its maximum working value H_o the flux density decreases from β_o along the curve as arrowed. The slope of this curve is the permeability which fixes the value of the inductance of the coil at any instant of time. We may therefore define "differential permeability" as

$$\mu = \frac{d\beta}{dH} \quad \ldots\ldots (29).$$

*Properties and Testing of Magnetic Materials, Thomas Spooner.

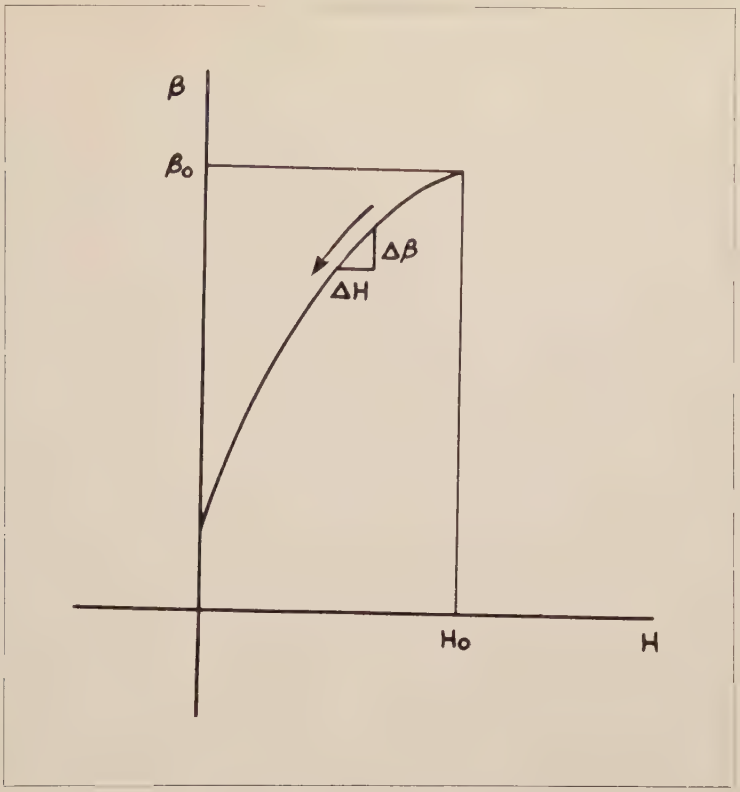


Fig. 16. Magnetism and Field Intensity in Electromagnet

The β-H Curve

The next step is the discovery of a relationship between apparent field intensity H and flux density β of the relay under test. Accordingly, a circuit was set up by means of which the current in a coil located in space B could be lowered by definite and known amounts, while a search coil and galvanometer measured the flux change from the maximum. The procedure was as follows:

1. Bring the iron to a cyclic state by repeated reversals.
2. Measure the initial flux by reversal.
3. From that initial maximum, drop the current by a small amount.
4. Measure by the galvanometer the corresponding drop in flux.
5. Bring the iron again to a cyclic state as before.
6. Measure that initial flux.
7. From that same initial current, drop the current by a little larger amount than "3."
8. Measure by a galvanometer the corresponding drop in flux.

Continue this process until the residual is reached by dropping the current from the initial maximum to zero.

Though it is a step-by-step process, there is no cumulative error, for each time the drop is taken from the initial state, which initial state could be duplicated within $\frac{1}{2}$ of 1%.

Table K—Magnetic Flux Decay in Collar Space as a Function of H.

1.	2.	3.	4.	5.	6.
I	*H*	*Net Deflection*	*Flux and*	*Flux Density*	
Amperes	*Gauss*	*θ in mm*	*Flux Change*	*Change*	*Value*
0.100	39.85	*43.19	*11696		12959
0.090	35.86	0.4	216.6	240	12719
0.080	31.88	1.3	704.1	780	12179
0.070	27.19	2.0	1083.2	1200	11759
0.060	23.91	3.0	1624.8	1800	11159
0.050	19.92	4.8	2599.6	2880	10079
0.040	15.94	6.2	3357.8	3721	9238
0.030	11.96	8.2	4441.0	4921	8038
0.020	7.97	11.1	6011.7	6661	6898
0.010	3.985	14.9	8069.7	8942	4017
zero	zero	19.6	10615	11762	1197

**Average of 11 readings, sandwiched between the other readings. The maximum deviation from this average was not over ½ of 1% from the average. This throw resulted from double flux cutting.*

Column 2, calculated from column 1 by

$$H=\frac{0.4\pi 6000}{18.92}I \quad \ldots\ldots (30).$$

Column 3 gives true deflection after applying zero correction.

Column 4 calculated by

$$\phi=10^8\frac{RK}{2n}\theta \quad \ldots\ldots (31).$$

$R=33.4\omega \quad K=3.243\times 10^{-6} \quad n=20$ turns in search coil.

Column 5 calculated by

$$\beta=\frac{\phi}{S}=\frac{\phi}{0.9025} \quad \ldots\ldots (32).$$

Column 6 obtained by subtracting each value of column 5 from the initial flux density 12959.

Columns 2 and 6 plotted in Fig. 17, as curve "1".

Table L—Smooth β-H Curve (from Table 10).

H	*β*	*H*	*β*
0	1197	20	10240
1	2040	22	10700
2	2800	24	11100
4	4100	26	11480
6	5200	28	11750
8	6240	30	12020
10	7130	32	12300
12	7920	34	12500
14	8650	36	12700
16	9280	38	12900
18	9780	40	13000

After trying on the above curve and its derivative the parabola, the hyperbola, the ellipse the function

$$\mu = \frac{\mu_0 K}{H^n + K}$$

the general form

$$\beta = k_1 + k_2 H + k_3 H^2 + k_4 H^3$$

and the exponential

$$\beta = K - U10^{-VH} \quad \ldots\ldots (33).$$

it was found that the latter gave the nearest fit to the β-H curve in the relay while decreasing from the given maximum value.

To fit the smooth curve, the following constants were calculated

$$K = 14212.3 \qquad U = 13015.3 \qquad V = 0.0257$$

or

$$\beta = 14212.3 - 13015.3 \times 10^{-0.0257H} \quad \ldots\ldots (34).$$

This fits curve 1 of Fig. 17 very closely, at most points not over $\frac{1}{8}$ of 1% off, the worst being only 1.7%.

Curve 2 of Fig. 17 is the decay curve from a lower point, $H = 20.44$ $\beta = 9652$, whose data are

Table M

H	β = *Flux Density*			
	Measured	*Smooth Curve*	*Math.*	*Deviation*
zero	1212.2	1212	1119	8.73%
2.39	3007.9	3000	3004	0.13%
3.98	3905.8	3950	4068	3%
5.98	5073.0	5073	5213	0.28%
7.97	6061.0	6061		
9.96	6958.6	6959		
11.96	7676.8	7700	7699	negligible
13.95	8260.5	8300	8288	0.145%
15.94	8754.2	8800		
17.93	9257.1	9240		
20.44	9652.2	9652	9651	negligible

The expression for the above is

$$\beta = 11520 - 10428 \times 10^{-0.036218H} \quad \ldots\ldots (35).$$

entered in the last column above, with deviations.

As a matter of additional interest, the above expression was applied to the apparent normal magnetization curve of a relay plotted as curve 3, Fig. 17. The data are given in Table *N*.

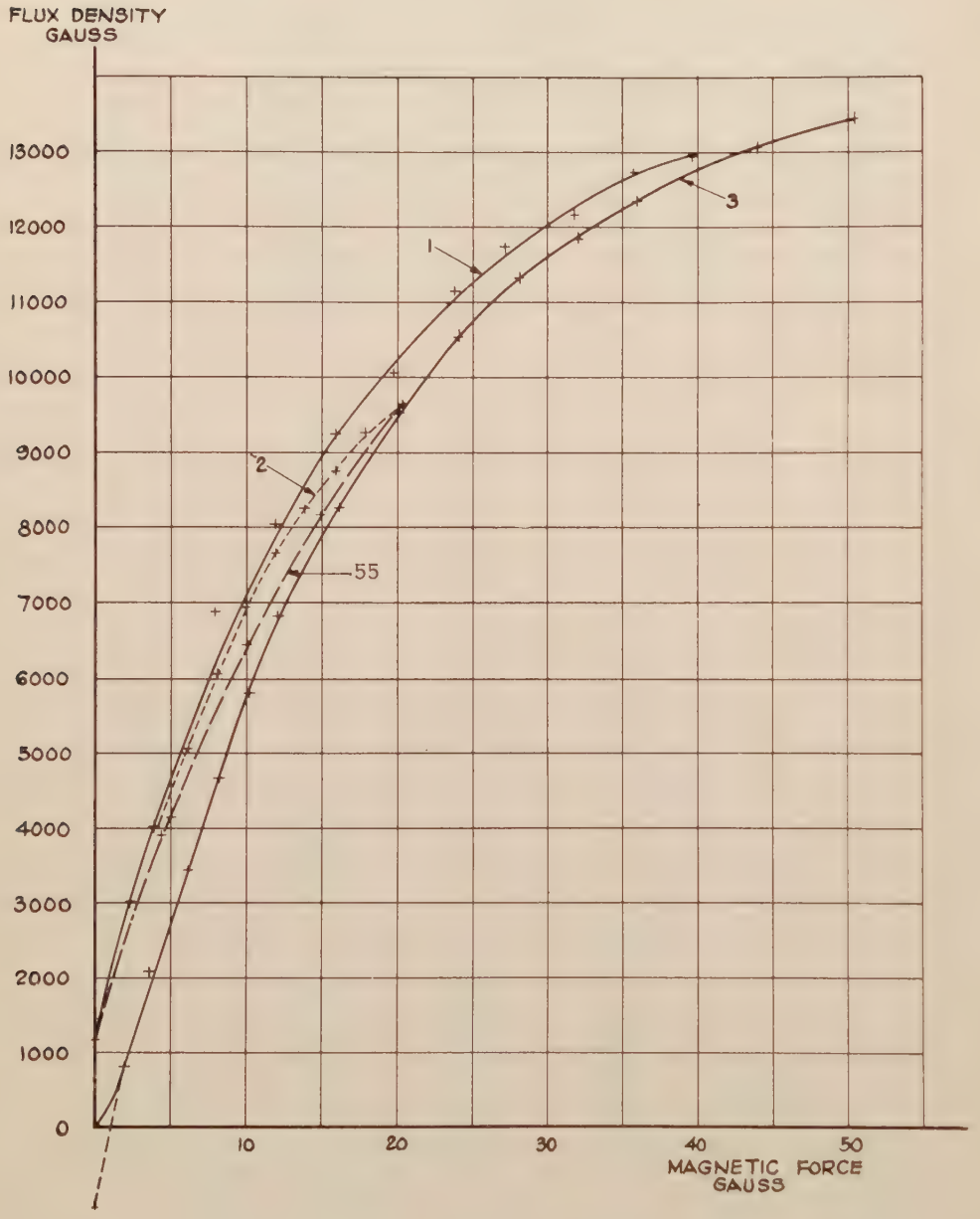


Fig. 17. Flux Density and Apparent Magnetic Force

Table N—Apparent Normal Magnetization Curve.

H	β	*Math.*	*Deviation*
zero	zero	−1078	
2.052	804.4	799.3	0.62%
4.085	2093		
6.117	3464		
8.150	4688		
10.16	5810	5812.2	0.046%
12.17	6818		
16.14	8283		
20.16	9523		
24.15	10523	10522.9	negligible
28.16	11327		
32.08	11859		
36.00	12341	12432	0.74%
39.95	12738		
43.96	13074		
47.88	13409	13388	0.172%

The expression for the above is

$$\beta = 14431.5 - 15309 \times 10^{-0.0255H} \quad \ldots\ldots (36).$$

The agreement is very close, everywhere except in the region of zero, where the expression is negative, while the curve must start at zero.

Hence the writer feels justified in assuming that the decay of magnetism in an electromagnet follows the β–H relationship of (33).

The permeability which conditions the inductance during decay of magnetism is defined by (29). Performing the indicated operation upon (33),

we have $$\frac{d\beta}{dH} = -U(-V)10^{-VH} \quad log_e 10 = \frac{UV}{log_{10} e} 10^{-VH}$$

so that

$$\mu = \frac{UV}{log\ e} 10^{-VH} \quad \ldots\ldots (37).$$

Derivation of t=funtion ϕ

We are now ready to derive an expression which relates the decaying magnetic flux to time, in terms of the constants of the structure.

Six conditions and six parameters appear.

$$\phi = S\beta \quad \ldots\ldots (38).$$

$$\beta = K - U10^{-VH} \quad \ldots\ldots (39).$$

$$H = \frac{0.4\pi N}{c} i \quad \ldots\ldots (40).$$

$$i = Ie^{-\frac{Rt}{L}} \quad \ldots\ldots (41).$$

$$L = 10^{-9}\frac{4\pi SN^2}{c}\,\mu \quad \ldots\ldots (42).$$

$$\mu = \frac{UV}{\log e}\,10^{-VH} \quad \ldots\ldots (43).$$

All attempts to secure a solution in the form $\phi = \text{funct } t$ failed because the quantities to be eliminated appeared as numbers and as logarithms in the same equation. It was necessary to solve for $t = \text{funct } \phi$ which was done by substitution, eliminating successively β, i, L, μ, and H. It resulted in

$$t = 10^{-9}\frac{4\pi VN^2}{cR(\log e)^2}\,(KS-\phi)\,\log\frac{\frac{0.4\pi NIV}{c}}{\log\frac{US}{KS-\phi}} \quad \ldots\ldots (44).$$

K, U, and V are constants of the core in (33).

S = cross section of core cm^2

c = length of magnetic circuit cm

N = number of turns in delay coil, unity in case of solid collar.

I = initial current in collar or delay coil necessary to support the initial magnetism.

R = Foucault resistance of collar and core.

It should be noticed that R, the Foucault resistance, appears in but one place, the denominator of the coefficient. Hence the time required to fall from the initial magnetism to any certain flux is inversely proportional to the resistance.

To make $t = o$,

$$\log\frac{\frac{0.4\pi NIV}{c}}{\log\frac{US}{KS-\phi}} = o$$

The only other variable $(KS-\phi)$ cannot be zero within the limits for which this expression is valid. From the above

$$\log\frac{US}{KS-\phi} = \frac{0.4\pi NIV}{c} \text{ or } \log(KS-\phi) = (\log US) - \frac{0.4\pi NIV}{c}$$

To make $t = \infty$,

$$\log\frac{\frac{0.4\pi NIV}{c}}{\log\frac{US}{KS-\phi}} = \infty$$

The number whose logarithm is infinite must itself be infinite. Since the numerator in question is a constant, the denominator must be zero. That is

$$\log \frac{US}{KS-\phi}=o \quad US=KS-\phi \text{ or } \phi=S(K-U)$$

When the attempt was made to put into this expression the previously measured constants of the relay which was being tested, the agreement was not satisfactory.

$K=11547$ $U=10428$ $V=0.036218$

$$t=0.0000423621\ (10397-\phi)\ \log \frac{0.740908}{\log \frac{9411.34}{10397-\phi}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (45).$$

The calculated values are in Table *P*.

Table P
Time

ϕ	*Curve*	*Calculated* (45)	*Calculated* (53)
8686	0	0.00013	0.000005
8350	0.005	0.0042	0.00394
8000	0.01	0.00975	0.00903
7800	0.015	0.01345	0.01177
7170	0.025	0.03588	0.023
5867	0.05	0.07061	0.055
4446	0.1	0.1439	0.109
3484	0.15	0.2175	0.163
2880	0.2	0.2803	0.210
2087	0.3	0.4002	0.300
1764	0.4	0.4739	0.357
1722	0.458	0.4855	0.366
1002	∞	∞	∞

The above is plotted in Fig. 14. It gives values of flux too high all along the curve, although practically correct at zero and infinity.

Retaining all the other constants, it was decided to calculate K, U, and V to fit the experimental curve. Accordingly, three points were selected and the above calculated to fit. These points were

$t=o$	$t_1=0.3$	$t=\infty$
$\phi_0=8686$	$\phi_1=2087$	$\phi\infty=1000$

These gave rise to three equations from the general equation, simplified thus

$$t=pV(KS-\phi)\ \log \frac{qV}{\log \frac{US}{KS-\phi}} \qquad \text{in which } p=10^{-9}\frac{4\pi N^2}{cR(\log e)^2},\ q=\frac{0.4\pi NI}{c} \quad \ldots\ldots\ldots\ldots\ldots\ldots (46).$$

When $t=o$ $\log \dfrac{qV}{\log \dfrac{US}{KS-\phi}}$ must be zero or

$$\frac{qV}{\log\frac{US}{KS-\phi}}=1 \text{ whence } qV=\log\frac{US}{KS-\phi_o} \quad \ldots\ldots(47).$$

at t_1 ϕ_1 we have $t_1=pV(KS-\phi_1)\log\frac{qV}{\log\frac{US}{KS-\phi_1}}$ $\ldots\ldots(48).$

at $t=\infty$

$$\log\frac{US}{KS-\phi_\infty}=o \text{ so that } US=KS-\phi_\infty \quad \ldots\ldots(49).$$

Solving (47), (48) and (49) for K, U, and V did not lead to explicit functions. They were of this form

K must satisfy

$$(KS-\phi_1)\log\frac{KS-\phi_\infty}{KS-\phi_o}\log\frac{\log\frac{KS-\phi_\infty}{KS-\phi_o}}{\log\frac{KS-\phi_\infty}{KS-\phi_1}}=10^8\frac{IR(\log e)^2}{N}t_1 \quad \ldots\ldots(50).$$

U must satisfy

$$(US+\phi_\infty-\phi_1)\log\frac{US}{US+\phi_\infty-\phi_o}\log\frac{\log\frac{US}{US+\phi_\infty-\phi_o}}{\log\frac{US}{US+\phi_\infty-\phi_1}}=10^8\frac{IR(\log e)^2}{N}t_1 \quad \ldots\ldots(51).$$

V comes out of (47) and (48) by the elimination of U and the use of the numerical value of K from (50)

$$V=\frac{c}{0.4\pi NI}\log\frac{KS-\phi_\infty}{KS-\phi_0} \quad \ldots\ldots(52).$$

The following constants were used in calculating K, U and V.

$c=18.92$ cm
$S=0.9025$ cm^2
$I=308$ amperes (from Fig. 8, to produce 8686 maxwells)
$R=3.0107\times10^{-6}$ ohm
$\phi_o=8686$ maxwells
$t_1=0.3$ second
$\phi_1=2087$ maxwells
$\phi_\infty=1000$ maxwells

As a result, $K=13789$
$U=12681$
$V=0.0236392$

so that the expression (44) becomes

$$t=0.0000276494\ (12444.5-\phi)\log\frac{0.483585}{\log\frac{11444.6}{12444.5-\phi}} \quad \ldots\ldots(53).$$

The resulting values are shown in Table P, column headed (53), and plotted in Fig. 14.

The agreement of (53) with the experimental curve is as good as the agreement of (17) and is more satisfactory because it has a rational basis, except for the empirical β-H expression.

A test of the truth of (44) is the conversion of (53) from the copper collar to other materials by merely changing the coefficient in whose denominator the Foucault resistance appears (and nowhere else). If we multiply this coefficient by the resistance which it contains and

divide it by the resistance to which the new curve must agree, a new coefficient will result which ought to fit the experimental curve for the same relay when equipped with the other collar.

For instance, the coefficient of (53) is

$$10^{-9}\frac{4\pi N^2 V}{cR(\log e)^2}=0.0000276494$$

To change to fit a collar with resistance R', multiply the coefficient by $\frac{R}{R'}$.

The joint Foucault resistances for the several collars used are worked out below:

Table Q.

	Resistance	*Factor*	*Coefficient*
Copper	3.0107×10^{-6}		0.0000276494
Coil	6.5861×10^{-6}	0.457132	0.0000126394
Coil (half)	11.3633×10^{-6}	0.264951	0.00000733762
Brass	10.8136×10^{-6}	0.278419	0.00000769815
Core	41.373×10^{-6}	0.07277	0.00000201206

The expression relating time and magnetic flux for the same relay equipped with a 282ω 6000-turn coil short circuited on itself should be

$$t=0.0000126394\ (12444.5-\phi)\log\frac{0.483585}{\log\frac{11444.6}{12444.5-\phi}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (54).$$

The others are similar, each having its own coefficient. Values of time were calculated from all of them, the results being given below:

Table R.

Magnetic Flux Maxwells	*Time*				
	Copper	*6000-T Coil*	*6000-T Coil half*	*Brass*	*Core alone*
8686	zero	zero	zero	zero	zero
8350	0.00394	0.001801	0.001044	0.001097	0.0002867
8000	0.009031	0.004128	0.002393	0.002514	0.0006572
7800	0.01177	0.005378	0.003117	0.003278	0.0008562
7170	0.02300	0.010511	0.006092	0.006402	0.001673
5867	0.05516	0.02522	0.01462	0.01536	0.004015
4446	0.1089	0.04980	0.02886	0.03033	0.007927
3484	0.1631	0.07454	0.04320	0.04540	0.01187
2880	0.2097	0.09585	0.05555	0.05838	0.01526
2087	0.3000	0.1371	0.07948	0.08353	0.02183
1764	0.3566	0.1630	0.09447	0.09928	0.02595
1722	0.3655	0.1671	0.09684	0.1018	0.02660
1500	0.4227	0.1931	0.1120	0.1177	0.03076
1200	0.5598	0.2559	0.1483	0.1559	0.04074
1100	0.6598	0.3016	0.1748	0.1837	0.04801

The above are plotted in Fig. 14.

Though not perfect, the agreement is close enough to indicate that the reasoning is sound as far as it goes. Undoubtedly, there are things going on in the electromagnet and copper collar which the writer has overlooked. But it is believed that a start has been made toward the desired solution of the electromagnet and that the facts discovered suggest the way for

further research. For instance, why do the experimental curves bend at the bottom more rapidly than the mathematical expression?

It is of interest to see what kind of β-H Curve is indicated by the constants K, U and V which the writer was forced to use to secure the above approximation (53).

The expression (33)
becomes $$\beta = 13789 - 12681 \times 10^{-0.023639H} \quad \ldots\ldots (55).$$
Plotted as (55) in Fig. 17, it appears less curved than the curve (2) whose expression (35) came from an experimental run on the relay. This difference of shape is additional indication of further research to be done.

Decay of Pole Flux.

Turning now to the magnetism at the pole, we find that it behaves somewhat differently, as will be seen in the copper curve in the lower part of Fig. 14. Immediately after the break of solenoid current there is a sudden drop radically different from the rest of the curve. This rest of the curve has a general form like that for the collar space, except that its apparent initial value is much lower than the experimental value. At the instant before the cut off, the solenoid is maintaining 8859 maxwells at the pole and 8686 maxwells in the collar. When the effect of the solenoid has been cut off, the current in the copper collar instantly rises to the value necessary to maintain 8686 maxwells. This corresponds to $NI = 308$ ampere turns (Fig. 8). But this has a power over the pole of only $P = 18.5$ so that it can produce at the pole only $PNI = 5698$ maxwells. This is approximately the value to which the pole flux falls immediately after the cut off. The curve is very steep for very small values of time and exact values are hard to secure.

Owing to this initial drop, a slight change becomes necessary in the expression (12) for the rest of the curve. The modified form is as follows:

$$\phi = \Phi + M10^{-Qt} - D10^{-kt} \sin \frac{\pi}{x}(t-y) \quad \ldots\ldots (56).$$

in which Φ = residual magnetic flux

M = computed constant

x = half period of sine, the time between two points

y = time assumed for first zero of sine.

$$Q = \frac{\log(\phi_y - \Phi) - \log(\phi_x - \Phi)}{(t_x - t_y)\log e} \quad \ldots\ldots (57).$$

$$M = \frac{t_x \log(\phi_y - \Phi) - t_y \log(\phi_x - \Phi)}{t_x - t_y} \quad \ldots\ldots (58).$$

$$k = \frac{\log\left[u \sin \frac{\pi}{x}(t_2 - y)\right] - \log\left[v \sin \frac{\pi}{x}(t_1 - y)\right]}{(t_2 - t_1)\log e} \quad \ldots\ldots (59).$$

$$D = \frac{u}{\sin \frac{\pi}{x}(t_1 - y)} e^{kt_1} \quad \ldots\ldots (60).$$

u = desired correction at arbitrary time t_1 flux ϕ_1

v = desired correction at arbitrary time t_2 flux ϕ_2

The time values y, t_1, t_2, and $(x+y)$ are in order of increase from zero. $t_x = x + y$

The following points were selected:

Residual flux $\Phi = 778$

$y = 0.05$ sec. $\quad \phi_y = 4400$

$t_1 = 0.1 \quad \phi_1 = 3216$

$t_2 = 0.2 \quad \phi_2 = 2102$

$x = 0.3$

$t_x = 0.35 \quad \phi_x = 1500$

Calculations for M and Q gave

$M = 4739 \quad Q = 2.33262$

These caused the first part of (56) to be

$$\phi = 778 + 4739 \times 10^{-2.33262t} \quad \ldots\ldots (61).$$

The errors produced by this at $t = 0.1$ and 0.2 second are

$u = 332$

$v = 295$ the expression being too large.

Putting these and the other necessary values into (59) and (60) gave $k = 3.52355 \quad D = 1495$ so that the complete expression is

$$\phi = 778 + 4739 \times 10^{-2.33262t} - 1495 \times 10^{-3.52355t} \; sin 10.4719(t - 0.05) \; . \; (62).$$

The results are given in Table S

Table S—Flux Decay at Pole by (62)

Time	*Flux*		
	Experimental	*Calculated*	*Error*
zero	8859	6265	
0 005	6208		
0.01	5960		
0.015	5558		
0.025	5092		
0.05	4428	4401	–0.6%
0.1	3216	3216	zero
0.15	2577		
0.2	2102	2102	zero
0.25		1846	
0.3	1631	1658	1.6%
0.35		1501	
0.4	1423	1360	–4.4%
0.458	1352		
0.5		1360	
∞	778	778	

The expression (44) was fitted to the experimental data for flux decay at pole with copper collar in the B space, by calculation of K, U, and V by expressions (50) (51) and (52) without any other change to take account of the relative locations of collar and flux considered.

The points at which the expression must be exact were

$t = 0 \quad \phi_0 = 6500$ (assumed from inspection of curve)

$t_1 = 0.282 \quad \phi_1 = 1701$ (chosen because it is the release flux)

$t = \infty \quad \phi_\infty = 778$

The same mechanical and electrical data were used as before; namely

$c = 18.92$ *cm*

$S = 0.9025$ *cm*^{2}

$R = 3.0107 \times 10^{-6}$ ohm

$I = 308$ amperes

The constants which resulted are

$K = 8556.4$

$U = 7694.4$

$V = 0.0368822$

The complete expression (44) becomes

$$t = 0.0000431389\ (7722.15 - \phi)\ log \frac{0.754495}{log \frac{6944.2}{7722.15 - \phi}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (63).$$

The values are given in Table *T*.

Expressions for other values of Foucault resistance were calculated by the factors of Table *Q*.

Table T—Flux Decay at Pole by Expression (44) (63)

Flux Maxwells	*Copper*	*6000-T Coil*	*Half Coil*	*Brass*	*Core*
8859					
6208	0.003732	0.001706	0.0009888	0.001039	0.0002716
5960	0.007808	0.003569	0.002069	0.002174	0.0005682
5558	0.01617	0.007392	0.004285	0.004502	0.001177
5092	0.02864	0.01315	0.007621	0.008008	0.002093
4428	0.05219	0.02359	0.01383	0.01453	0.003798
3216	0.1174	0.04674	0.03110	0.03269	0.008543
2577	0.1693	0.07741	0.04487	0.04715	0.01232
2102	0.2217	0.1014	0.05874	0.06173	0.01613
1631	0.2949	0.1348	0.07814	0.08211	0.02146
1423	0.3399	0.1554	0.09006	0.09464	0.02474
1352	0.3583	0.1638	0.09494	0.09976	0.02607
1200	0.4059	0.1855	0.1075	0.1130	0.02954
1100	0.4446	0.2040	0.1183	0.1243	0.03250
1000	0.5011	0.2291	0.1328	0.1395	0.03647
900	0.5860	0.2679	0.1553	0.1632	0.04262
800	0.8174	0.3737	0.2166	0.2276	0.05948

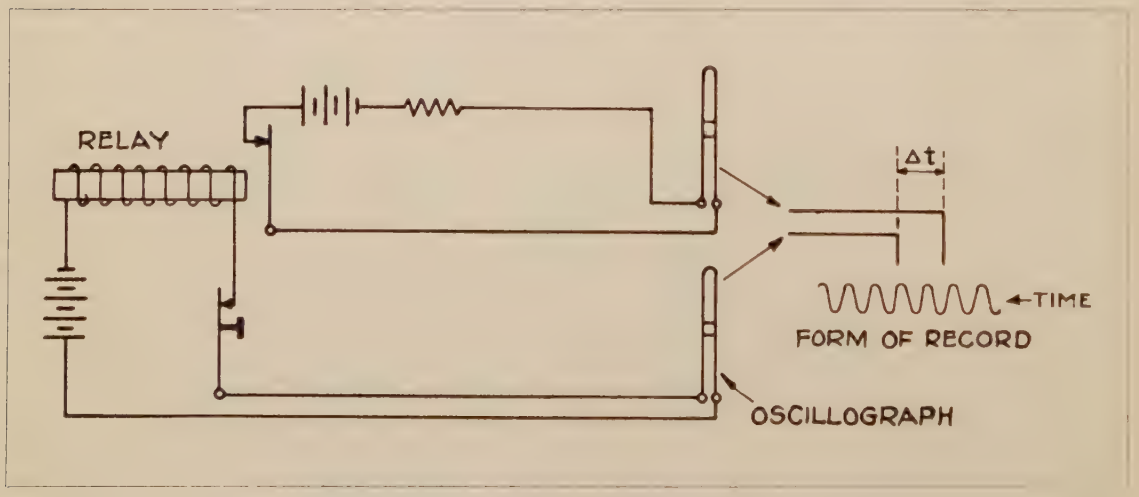


Fig. 18. Measurement of Release Time

It is to be noted again that the experimental curves bend more rapidly at the bottom than the graph of the mathematical expression.

RELEASE TIME OF ARMATURE

The time actually required for the armature to start to fall away after the current has been cut off from the solenoid was measured by means of an oscillograph. The connections were arranged as in Fig. 18 and the oscillograph in Fig 19. The relay contact shown was a special one, mounted on the armature lever, so that there was as little movement of the armature as possible before breaking the circuit. The switch in the solenoid circuit was opened by hand, just after the oscillograph shutter opened

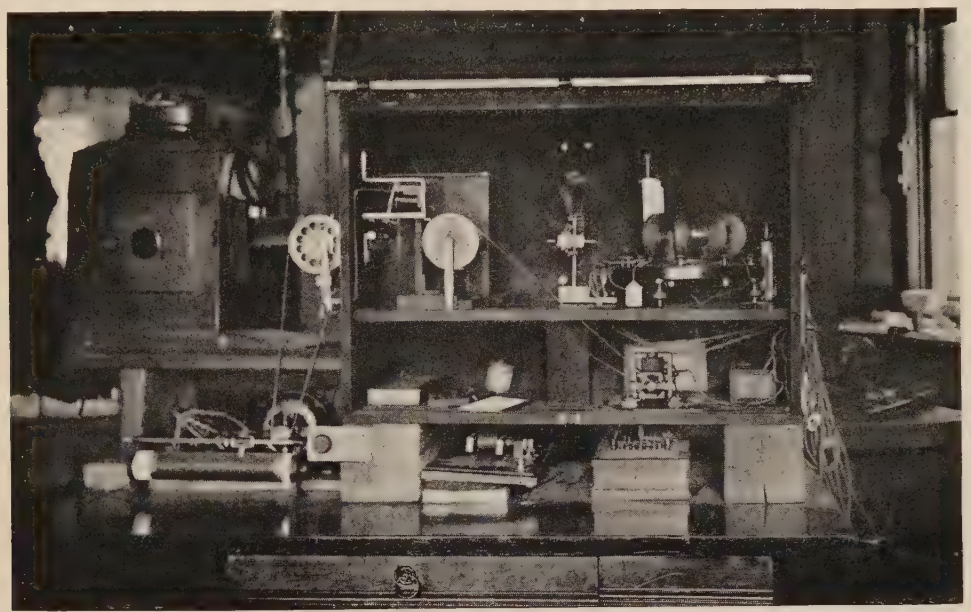


Fig. 19. Oscillograph

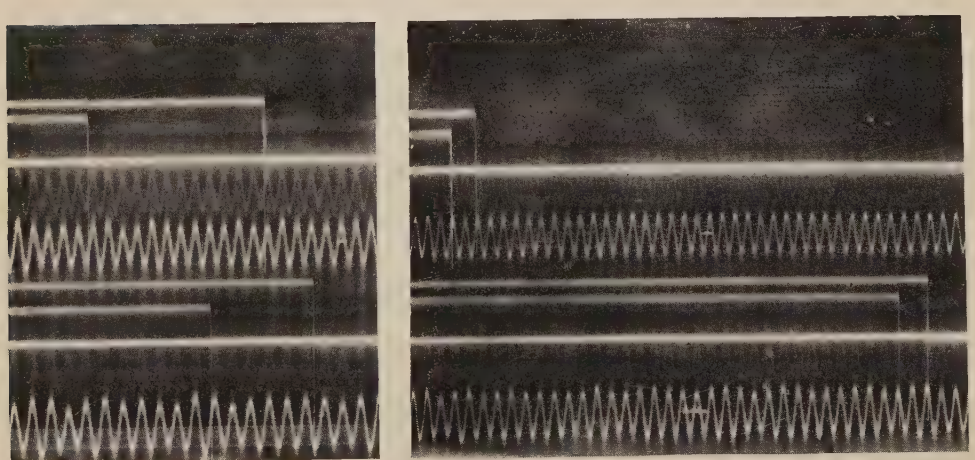


Fig. 20. Oscillograms of Release Time

A 100-cycle tuning fork furnished timing current to a separate galvanometer. Thus we have the possibility of recording on a single film two events, the instant of break of the solenoid current and the instant of release of the armature, together with a time-scale. Two sample records are shown in Fig. 20.

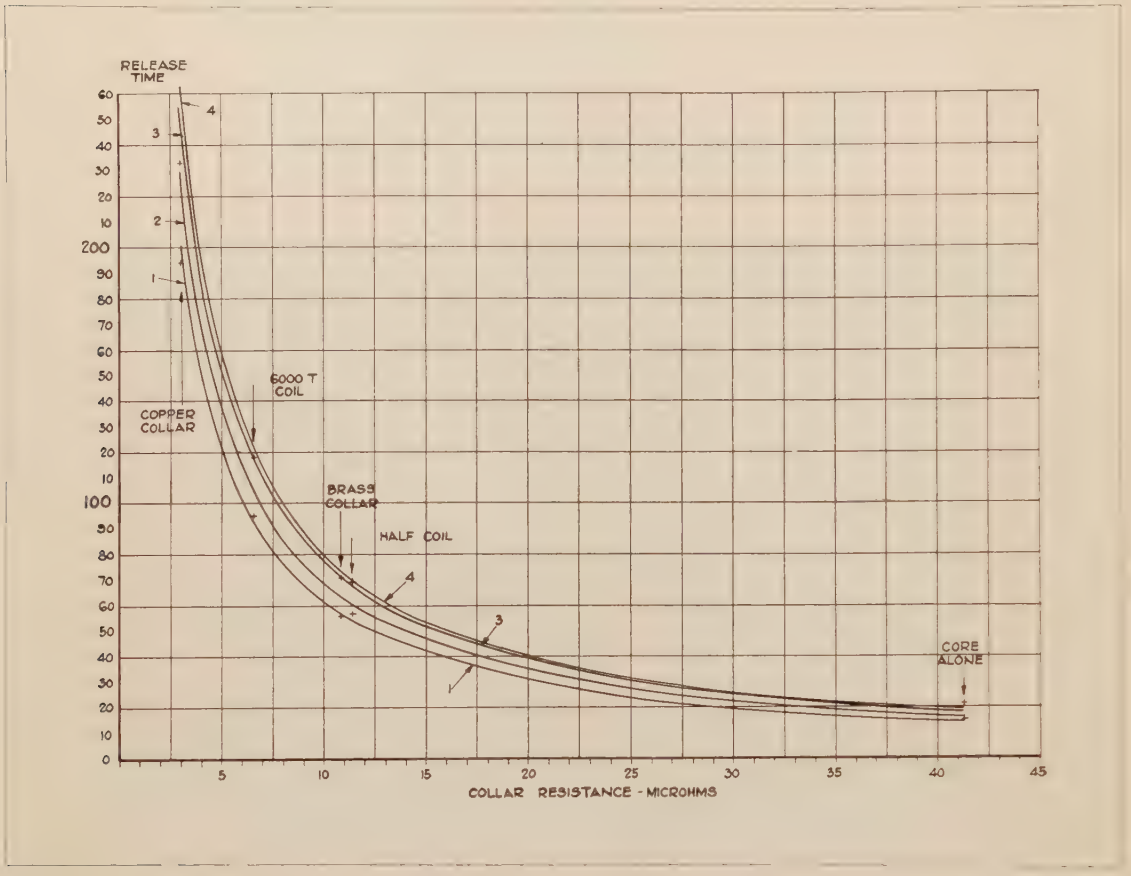


Fig. 21. Release Time of Armature and Collar Resistance

The results are given in the table following, where the release time is expressed as a function of the Foucault resistance. The Foucault resistance is computed from expression (8) for collars and (9) for coils, corrected for the Foucault resistance of the core (Table *T* and following).

"Release time" values for the "6000-T half" coil were computed from the flux decay data, Fig. 14, and are here inserted for the purpose of comparison.

Table U—Armature Release Time (in milli-seconds)

	Foucault Resistance Microhms	1 *For* $NI = 307$	2 *For* $NI = 408$	3 *For* $NI = 508$	4 *For* $NI = 609$
Copper	3.0107	193.7	218.2	233.2	244.7
6000-T	6.58608	95.2	110.3	118.1	120.5
Brass	10.8136	55.8	63.8	70.8	73.5
6000-T half	11.3633	56.9	66.5	69	67.6
Core	41.373	15.09	19.68	20.75	21.89

The data of curves 1 and 3 of Table *U* are indicated by cross marks on Fig. 21. Each series is closely represented by a rectangular hyperbola of the form

$$t=\frac{F}{R} \qquad (64).$$

in which t = release time in seconds

R = joint Foucault resistance of collar and core

F = constant whose meaning will be discussed below.

Expression (64) may be passed at will through only one finite point.

If the collar resistance could be zero, the decay of flux would set up a current which would never stop and would maintain a magnetic field forever. Hence for $R=0$ $\quad t=\infty$

If the joint Foucault resistance—made up of core and collar—could be infinite, the magnetic field would have nothing to support it, and it would instantly drop to the residual value, passing through the release flux on the way down, making the release time zero. The movement of the armature would still occupy a finite interval of time, but that is beyond the scope of this particular expression.

Taking the Foucault resistance of the brass collar as the single finite point, the constant F was calculated for each interval condition (ampere-turns in solenoid). Table *V* gives the resulting expressions and values.

Table V—Release Time Expressions

Foucault Resistance Microhms		1 $NI=307$ $t=\frac{6.034\times10^{-7}}{R}$		2 $NI=408$ $t=\frac{6.899\times10^{-7}}{R}$		3 $NI=508$ $t=\frac{7.656\times10^{-7}}{R}$		4 $NI=609$ $t=\frac{7.948\times10^{-7}}{R}$	
		Exp.	*Calc.*	*Exp.*	*Calc.*	*Exp.*	*Calc.*	*Exp.*	*Calc.*
Copper	3.0107	0.1937	0.2004	0.2182	0.2292	0.2332	0.2543	0.2447	0.2640
	4.		0.1508		0.1725		0.1914		0.1987
	5.		0.1207		0.1380		0.1531		0.1590
Coil	6.58608	0.0952	0.0916	0.1103	0.1048	0.1181	0.1162	0.1205	0.1207
Brass	10.8136	0.0558	0.0558	0.0638	0.0638	0.0708	0.0708	0.0735	0.0735
Half-C	11.3633	0.0569	0.0531	0.0665	0.0607	0.0690	0.0674	0.0675	0.0699
	25.		0.0241		0.0276		0.0306		0.0318
Core	41.373	0.0151	0.0146	0.0197	0.0167	0.0208	0.0185	0.0219	0.0192

Exp. = experimental result

Calc. = results of mathematical expressions

These were plotted as curves in Fig. 21. The corresponding numbers apply. Curve 4 was not plotted to its end because it lies too close to Curve 3, in which we have more interest because the decay of magnetic flux, Fig. 14, was taken from 508 ampere-turns in the solenoid.

It is to be noted that the agreement of mathematical expression with experimental results is very close.

If we compare expression (64) with (44) for a given magnetic flux, the reasonableness of (64) is evident. If we insert in (44) the values which apply to the flux decay at the pole and the value of release flux (1701 maxwells), we obtain an expression which has only two variables, t and R. It is then in the form of (64).

According to this reasoning

$$F=10^{-9}\,\frac{4\pi N^2 V}{c(\log e)^2}(KS-\phi_r)\,\log\frac{\frac{0.4\pi NIV}{c}}{\log\frac{US}{KS-\phi_r}} \qquad (65).$$

in which ϕ_r = release flux

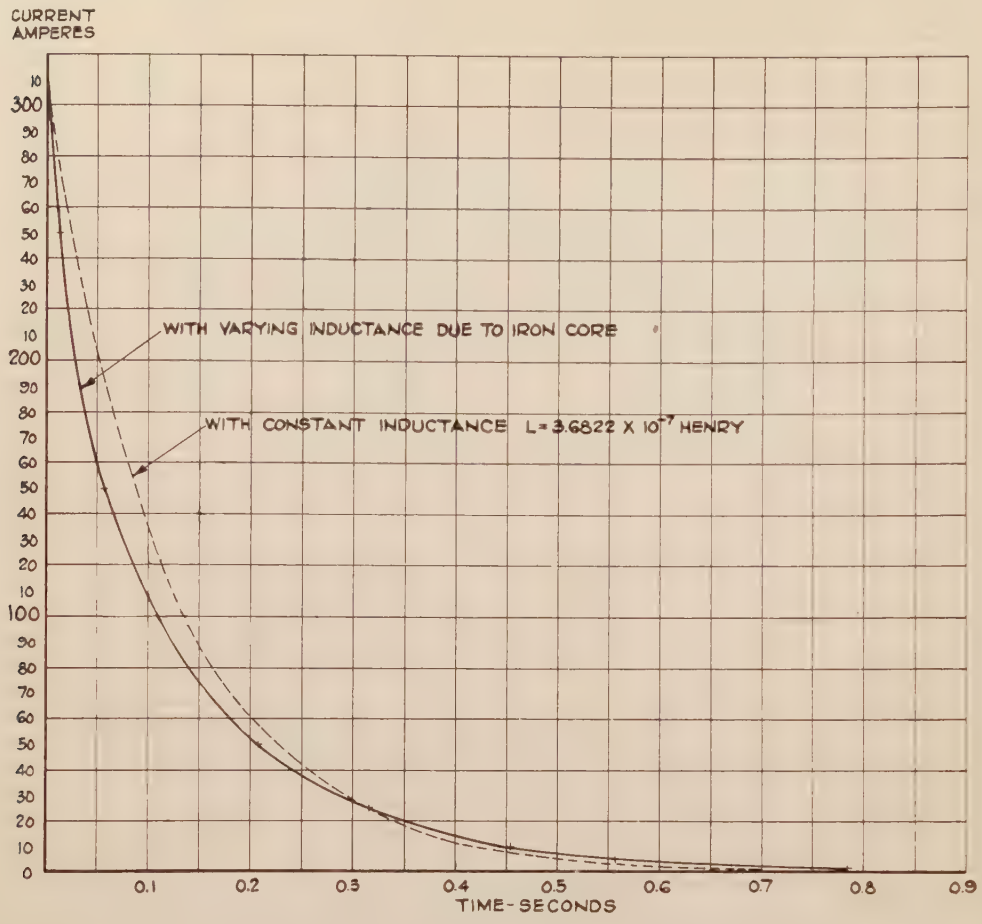


Fig. 22. Decay of Current in Copper Collars

CURRENT IN COPPER COLLAR

It was not found to be possible to measure the eddy current in the copper collar with the means at hand. It was, however, calculated in the following manner:

From (18) solving for t we have

$$t = \frac{L}{R \log e} \log \frac{I}{i} \quad \ldots\ldots (66).$$

We consider also

$$L = 10^{-9} \frac{4\pi S N^2}{c} \mu \quad \ldots\ldots (42).$$

$$\mu = \frac{UV}{\log e} 10^{-VH} \quad \ldots\ldots (43).$$

$$H = \frac{0.4\pi N}{c} i \quad \ldots\ldots (40).$$

Eliminating H, μ and L by substitution, we secure

$$t = 10^{-9} \frac{4\pi N^2 SUV}{cR\ (log\ e)^2} 10^{-\frac{0.4\pi NV}{c} i}\ log \frac{I}{i} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (67).$$

Apparently it could not be solved as a function of time.

At $i = I$ $log \frac{I}{i} = o$ so that $t = o$

At $i = o$ $log \frac{I}{i} = \infty$ so that $t = \infty$

Using $R = 3.0107 \times 10^{-6}$ ohm and the rest of the known constants the expression for decay of current in the copper collar became

$$t = 0.31644 \times 10^{-0.0015701\ i}\ (2.48855 - log\ i) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (68).$$

which gives Table W

Table W—Current in Copper Collar

Variable Inductance		*Constant Inductance*	
Time	*Current*	*Time*	*Current*
Zero	308	Zero	308
0.001223	300	0.05	205
0.01173	250	0.1	136
0.0288	200	0.2	60.3
0.05749	150	0.3	26.5
0.1077	100	0.4	11.7
0.2085	50	0.5	5.16
0.3153	25	0.6	2.28
0.4543	10	0.7	1.007
0.5561	5	0.8	0.444
0.7846	1	∞	zero
∞	zero		

These values are plotted in Fig. 22

Compare this with the decay curve with constant inductance

$$L = 3.6822 \times 10^{-7} \text{ henry}$$

whose expression is

$$i = 308 \times 10^{-3.551\ t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (69).$$

Evidently the inductance is much lower at the start and increases as the magnetism decreases.

SUMMARY OF RESULTS

1. There is a hyperbolic relationship between the release time (t) of a relay and the Foucault resistance (R) of the magnet.

$$t = \frac{F}{R}$$

in which F is a constant which involves the relay structure, condition at release, and the magnetic flux at which the armature releases.

This constant has the form

$$F = 10^{-9}\frac{4\pi N^2 V}{c(\log e)^2}(KS-\phi_r)\ \log\frac{\frac{0.4\pi NIV}{c}}{\log\frac{US}{KS-\phi_r}}$$

ϕ_r = release flux in maxwells. Common logarithms.
The other symbols are defined below.

The shape of this curve explains why a slow relay which must operate many springs is very sensitive to slight impurities in the copper. The impurities may raise the resistivity only a little, yet this makes a large reduction of release time.

2. A copper collar on the iron core of a relay causes within itself, during the decay period, a magnetic flux curve which is approximately represented by the mathematical expression

$$t = 10^{9}\frac{4\pi N^2 V}{cR(\log e)^2}(KS-\phi)\log\frac{\frac{0.4\pi NIV}{c}}{\log\frac{US}{KS-\phi}}$$

in which

t = time in seconds
ϕ = flux in maxwells } the variables
S = cross sectional area of core (cm^2)
c = length of magnetic circuit (cm)
R = Foucault resistance of collar and core (jointly) ohms
N = number of turns in collar (unity if made of solid metal)
I = initial current in collar (amperes)
K, U, V } constants in the β-H curve given below.

An alternative expression is

$$\phi = \Phi + M10^{-Qt} - D10^{-kt}\ \sin\frac{\pi}{x}t$$

in which Φ = residual flux
$\Phi + M$ = initial flux
Q, D, k and x are empirical constants.

3. The decay part of the hysteresis loop of an electromagnet is closely represented by the empirical expression.

$$\beta = K - U10^{-VH}$$

in which K, U and V are empirical constants.

4. The permeability which is involved in the flux decay process of a relay is the "differential permeability" defined by $\mu = \frac{d\beta}{dH}$

and is for this case $\mu = \frac{UV}{\log e}10^{-VH}$

5. The magnetic flux decay at the pole of an electromagnet is a curve of two parts.

(a) A quick change from the initial value to that which the Foucault current in the copper collar can maintain at that time. (b) A gradual decay similar to (2) above, but with different constants.

The expression of (2) above applies to the time (b) but with slightly different values of K, U, and V.

The alternative expression has a slight change to enable it to apply to (b).

$$\phi = \Phi + M10^{-Qt} - D10^{-kt} \sin \frac{\pi}{x}(t-y)$$

in which all constants are as before. A new constant "y" is added. All are empirical.

In this connection, the relation

$$P = \frac{\phi}{NI}$$

is very useful. ϕ = flux at pole produced by NI ampere-turns in the copper collar. It gives a measure of the computed constant M, for the pole flux quickly changes from the initial value to that which the collar Foucault current can cause at the pole $(\Phi + M)$.

6. Practically all of the longitudinal flux in the case considered is in the core. Leakage flux is perpendicular to the axis.

7. The decay of current in the copper collar follows the expression.

$$t = 10^{-9} \frac{4\pi N^2 SUV}{cR(\log e)^2}(\log I - \log i)10^{-\frac{0.4\pi NV}{c}i}$$

The symbols are defined above.

The research work for this thesis yielded a mass of data which has not yet been exhausted. In addition, many lines of investigation have suggested themselves. It is the hope of the writer to continue the work and to make it of general application to electromagnets of any form.

Expression of thanks is here made to Prof. Henry Crew and the staff of the Physics Department of Northwestern University for suggestions of a very helpful nature; also to Mr. Harold C. Pye and Mr. Henry G. Meltvedt of the research department, Automatic Electric Inc., for valuable assistance in numerical computations.

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BIBLIOGRAPHY

Benedict, F. G., and Cady, W. G., Magnetic Reaction Produced by a Copper Disc Rotating Between the Poles of a Magnet. Phys. Rev. Ithaca, N. Y., 34 1912 (315-316).

Berg, E. J., Electrical Engineering, advanced course (a book).

Defretin, Sur les courants de Foucault dans un tore de fer doux plein de l'influence de l'hysteresis. C.R. 158-1924 p. 1885.

General Electric Review, Short circuited windings on direct current solenoids. Aug. 1918.

Rayleigh, Self induction of electrical currents in a thin anchor ring. London Proc. Roy. Soc. Series A, 86-1912 (562-571).

Wells, R. T. Inductance of currents in cylindrical cores. Phys. Rev. N.Y. 26 1908 (337-357).

Wilson, Ernest, Distribution of magnetism as affected by inducted currents in an iron cylinder when rotated in a magnetic field. Proc. Roy. Soc. London, Vol. 69, No. 457, March 21, 1902, pp. 435-449.